

Vector Analysis, Homeworkz 3

September 2013

1. Evaluate the triple integral $\iiint_{\mathbf{W}} \frac{dV}{\sqrt{x^2+y^2+z^2}}$ where \mathbf{W} is the solid region between the upper hemispheres of two concentric spheres of radii $a < b$.
2. Consider the vector field $\mathbf{F}(x, y) = (-2y, 2x)$.
 - a. Show that the parameterized curve $(\cos(2t), \sin(2t))$ is a flow line of this vector field. Show that the family of parameterized curves $(a \cos(2t), a \sin(2t))$ are flow lines of \mathbf{F} . What curve does this parameterization describe?
 - b. Find a vector field $\mathbf{G}(x, y)$ which is everywhere perpendicular to the field \mathbf{F} .
 - c. Find the flow line of the vector field \mathbf{G} which goes through the point (x, y) , $x^2 + y^2 > 0$, at $t = 0$.
3. Using spherical coordinates, find the volume of the spherical cap: the solid region $x^2 + y^2 + z^2 \leq 10$ which lies above the plane $z = 1$.