

Vector Analysis, Homeworkz 5

October 2013

1. Consider the vector field $\vec{F} = 3\vec{i} - 2\vec{j} + \vec{k}$.
 - a) Show that \vec{F} is a gradient field.
 - b) Describe the equipotential surfaces of \vec{F} in words and with sketches
 - c) Calculate the work done by the vector field \vec{F} in moving a particle along the parameterized path $\vec{r}(t) = (2, -5, 7) + t(-2, -4, 6)$, $-1 \leq t \leq 1$.

2. For the parameterized helix \mathcal{C} , given by $\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + t\vec{k}$, on the time interval $0 \leq t \leq 1.25\pi$, calculate the path integral

$$\int_{\mathcal{C}} yz^2 e^{xyz^2} dx + xz^2 e^{xyz^2} dy + 2xyz e^{xyz^2} dz.$$

3. Consider the vector field $\vec{F} : \mathbb{R} \times (0, +\infty) \rightarrow \mathbb{R}^2$ given by

$$\vec{F}(x, y) = \frac{x + xy^2}{y^2} \vec{i} - \frac{x^2 + 1}{y^3} \vec{j}.$$

- a) Determine whether \vec{F} is a gradient field or not, and give an explanation of your conclusion.
- b) Calculate the work done in moving a particle along the curve $y = 1 + x - x^2$ from $(0, 1)$ to $(1, 1)$.