Math 231, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Homework 3, Due Thursday October 13

1. Consider the differential equation and intitial value

$$bxydx + \left(3x^2 + 4\cos(y)\sin(y)\right) dy = 0, \quad y(1) = \frac{\pi}{4}$$

a) Find a value of the parameter b, which makes the equation exact.

b) For this value of b, find the general solution, and the solution of the initial value problem in implicit form.

c) What conclusion can be made concerning the interval of existence.

2. Consider the differential equation

$$\left(3yx^2 + 2xy + y^3\right) dx + \left(x^2 + y^2\right) dy = 0$$

a) Show that there is an integrating factor of the form $u = u(x)$, and find this function which makes the equation exact.

b) Using the integrating factor found in part a), integrate the equation to find the general solution $F(x, y) = c$.

c) Find all of the critical points of the function $F(x, y)$ you found in part b).

3. Consider the coupled pair of first order nonlinear equations

$$\frac{dx}{dt} = y^2 - x^2, \quad \frac{dy}{dt} = -2xy$$

a) Show that by introducing a complex variable $q = x + iy, i^2 = -1$ the system of differential equations can be expressed in terms of q alone. Integrate this equation to find explicit solutions in $x(t), y(t)$. Hint: calculate q^2.

b) show that these solutions from part b) must lie on circles in the $x - y$ plane by finding a conservation law for the system in the form of a function $F(x, y)$ which must be constant along the solutions. Hint: reparameterise the solutions, eliminating the time variable and look for an integrating factor for the resulting scalar differential equation.