Math 231, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Homework 5, Due Thursday November 10

1 Find the general solution using variation of parameters

$$D^2y + y = \tan(t), -\pi/2 < t < +\pi/2$$

Hint: use a standard table of integrals to evaluate the integrals you need.

2. A spring is streched 10cm by a force of 3 N. A mass of 2kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3N when the velocity of the mass is 5 m/sec. If the mass is pulled down 5cm below its equilibrium position and given an intial upward velocity of 10cm/sec, determine its position y at any time t in phase amplitude form. Find the quasi frequency of the resulting motion.

3. Find the general solution for $t > 0$ using variation of parameters (look for fundamental set of homogeneous solutions t^r).

$$t^2y'' + 7ty' + 5y = t$$

4. Find the general solution and describe the behaviour of the solution as $t \to \infty$. Also sketch some representative solution trajectories in the plane, using eigensolu-

tions.

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix} \mathbf{x}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ \\ x_2 \end{pmatrix}$$