Math 231, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Homework 6, Due Thursday, November 24

1) Find the general solution and describe the behaviour of the solution as $t \to \pm\infty$.

$$x' = \begin{pmatrix} 0 & 1 \\ -4 & -4 \end{pmatrix} x$$

Show that the origin in the plane is globally asymptotically stable. Also sketch the phase plane for solution trajectories in the plane. Find a spring mass problem which is equivalent to this matrix differential equation.

2) Find the general solution and describe the behaviour of the solution as $t \to \infty$. Show that there is a two dimensional invariant plane of solutions which are expanding and a one dimensional invariant line of solutions which are contracting as $t \to \infty$. Sketch some representative solution trajectories in three dimensional space.

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

3) Find the matrix exponential e^{At}. Hint: use the block diagonal structure of A.

$$x' = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{pmatrix} x$$

4) Find a fundamental set of generalized eigensolutions for the matrix differential equation

$$x' = \begin{pmatrix} 2 & -1 & -4 \\ 0 & 2 & -4 \\ 0 & 1 & -2 \end{pmatrix} x$$