Math 231, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Tutorial , Monday, November 28

1) We will examine four related questions for the mathematical pendulum with friction. The governing equation for the pendulum with friction

$$y'' + by' + k \sin y = 0, \quad b, k > 0$$

a) Write the pendulum equation as a pair of coupled first order equations.

b) Determine all equilibrium points for the planar vector field.

c) Calculate the linear part of the vector field at each of the equilibrium points $(0, 0)$ and $(\pi, 0)$.

d) For parameter values $b = 4, k = 2$ sketch the solution trajectories in a neighborhood of each equilibrium point $(0, 0)$ and $(\pi, 0)$.

e) For parameter values $b = 2, k = 2$ sketch the solution trajectories in a neighborhood of each equilibrium point $(0, 0)$ and $(\pi, 0)$.

Can we determine the region of the phase plane which is asymptotic to the equilibrium at $(0, 0)$