Math 231, Introduction to Differential Equations, Fall 2010

Queen's University, Department of Mathematics

Tutorial , Monday, September 20

1) Find the general solution and use it to determine how solutions behave as $t \to \pm\infty$

$$(1+t^2)\frac{dy}{dt} + 4ty = (1+t^2)^{-2}$$

2. Solve the intial value problem and use this to answer the questions below

$$ty' + (t+1)y = 2te^{-t}, \quad y(1) = a, \quad t > 0.$$

How do the solutions appear to behave as $t \to 0$, does the behavior depend on the choice of initial condition a. Let a_0 denote the value of the parameter a which signals a qualitative change in the behaviour of the solutions as $t \to 0$. Find the value of a_0 and describe the qualitative change.

3. Find the solution of the initial value problem , sketch the plot of the solution, determine at least approximately (or graphically) what the interval of existence for the solution might be

$$y' = \frac{2x}{y + x^2 y}, \quad y(0) = -2$$

$$y' = \frac{1+3x^2}{3y^2 - 6y}, \quad y(0) = 1$$

4. Consider a tank used in certain hydrodynamic experiments. After one such experiment the tank contains 200L of a dye solution with a concentration of 1g per L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2L per min., the well stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1 percent of its original value.