Math 231, Introduction to Differential Equations, Fall 2010

Queen's University, Department of Mathematics

Tutorial , Monday, September 27

1) Determine the asymptotic behavior of solutions to the initial value problem with initial condition $-\frac{\pi}{4} \leq y_0 \leq \frac{\pi}{4}$

$$\frac{dy}{dt} = y\sin(y) - \sin(2y)$$

2. For the same differential equation as in previous question, show that $y = \pi$ is an attracting equilibrium point. Why does this imply that there must be another equilibrium point between $\frac{\pi}{4}$ and $\frac{\pi}{2}$. What is the stability type of this additional equilbrium point.

3. A population of field mice has a natural logistic population growth law

$$y' = ay - y^2, \quad \text{mice per month.}$$

This population of field mice is preyed upon by a community of coyotes. The coyotes eat the field mice at a rate which is proportial to the population y(t). Suppose this rate is $-ey$ measured in mice per month. As the parameter e varies, the population of field mice can have different long term behaviour. Express this long term behaviour as a bifurcation diagram in the $e - y$ plane.