**Math 231, Introduction to Differential Equations, Fall 2011**

**Queen's University, Department of Mathematics**

**Tutorial , Friday, October 15**

**1 )** Find a fundamental solution set for the second order linear homogeneous equation

$$\left(2D^2 + 3D - 2\right)[y] = 0, t \in R, D = \frac{d}{dt}$$

Use this solution to graph the solution of the IVP $y(0) = 1, y'(0) = 3$.

**2.** Show that a family of exponential functions forms a fundamental solution set for the third order linear homogeneous equation by factoring the differential operator into a composition of three first order operators

$$\left(D^3 + 2D^2 - D + 2\right)[y] = 0, \quad D = \frac{d}{dt}$$

**3.** Consider the second order linear homogeneous differential equation

$$2y'' + y' - 4y = 0.$$

a) show that there are solutions $y_1(t), y_2(t)$ with the following asymptotic properties

$$\lim_{t\to\infty} y_1(t) = 0, \quad \lim_{t\to-\infty} y_2(t) = 0$$

b) solve the initial value problem $y(0) = 0, \quad y'(0) = 1$. Comment on the asymptotic properties of this solution as $t \to \pm\infty$, and sketch the graph of the this solution.

**4.** Prove that if $y_1, y_2$ are a FS of solutions to the linear second order equation

$$\left(D^2 + p(t)D + q(t)\right)[y] = 0, \quad t \in I$$

where $p(t)$ and $q(t)$ are continuous on the interval I, then it is not possible for both $y_1$ and $y_2$ to have a simultaneous critcal point at $t_0 \in I$.