Math 231, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Tutorial , Monday October 23

1) Find the general solution to the nonhomogeneous differential equation

$$L[y] = (D^2 - 4)[y] = -3e^3t + 10te^3t, \quad D = \frac{d}{dt}$$

Then find the solution to the initial value problem $y(0) = 1, y'(0) = 0$.

2. Suppose we consider a simple mechanical system where we attach a 4kg mass to a dampened spring system. The mass is stretched 1 meter from its equilibrium position and released with 0 velocity. We assume that the spring constant is $k = 1\frac{\mathrm{N}}{\mathrm{m}}$, and the damping constant is $b = 4\frac{\mathrm{N}}{\mathrm{m/s}}$

a) Using Newton's second law of motion, write the differential equation and the initial values for $y(t)$ the displacement from equilibrium

b) Find the resulting motion of the mass (displacement from equilibrium as a function of time) corresponding to the initial value problem above.

3 Prove that the exponential shift theorem we talked about in class

$$P(D)[e^{\lambda t}y] = e^{\lambda t}P(D+\lambda)[y], \quad D = \frac{d}{dt}$$

holds when $P(D) = (D-\alpha)$ or $P(D) = (D-\alpha)(D-\beta)$. Generalise your argument to any

$$P(D) = (D - a_1)^{m_1} \cdots (D - a_k)^{m_k}, \quad a_i \in R, m_i \in N$$