Math 231, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Tutorial , Monday November 7

1) Find two linearly independent vector eigen solutions for the system of differential equations

$$\frac{dx}{dt} = Ax, \quad A = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix}$$

Show that this system is equivalent to the scalar second order equation $(D^2 + 2D - 3)[x] = 0$, and that the eigensolutions correspond to exponential solutions found by using the characteristic polynomial. Sketch the eigensolutions in the $x_1 - x_2$ plane.

2. Find three linearly independent eigensolutions

$$\frac{dx}{dt} = Ax, \quad A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Hint: calculate the eigenvalues and a basis of eigenvectors for the matrix A.

3. Consider the spring mass system where a mass of 1gm is attached to a spring and hung vertically. The weight of the mass stretches the spring 9.8 cm. Find the solution to the initial value problem assuming no damping when the displacement from the equilibrium configuration is 2cm and the initial velocity of the mass is -4cm.

Using the same parameters and initial conditions, find the solution when the damping coefficient is b=12 dynes per cm per second. Can you explain why the quasifrequency decreases with damping. Dyne is the unit of force in the gram-cm-second system of units.