Math 231, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Tutorial , Monday November 14

1) Find the general solution or matrix exponential, indicate the behavior of solutions as $t \to \pm\infty$, and draw the trajectories in the phase plane. Eigenvalues of A are $\lambda_1 = -\frac{2}{4}, \lambda_2 = -2$

$$\begin{pmatrix} x' \\ \\ y' \end{pmatrix} = \begin{pmatrix} \frac{-5}{4} & \frac{-3}{4} \\ \\ \frac{-3}{4} & \frac{-5}{4} \end{pmatrix} \begin{pmatrix} x \\ \\ y \end{pmatrix}$$

Then find the solution to the initial value problem $x(0) = 1, y(0) = -2$.

2. Using geometric reasoning, draw the phase portrait for the system

$$\begin{pmatrix} x' \\ \\ y' \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix} \begin{pmatrix} x \\ \\ y \end{pmatrix}$$

.Hint: What operation on the tangent vectors to the trajectories from question 1, has been done in question 2.

3 Find three linearly independent eigensolutions for the system of equations

$$\begin{aligned} x' &= -x + 2y + 2w \\ y' &= 2y + w \\ w' &= -y + 2w \end{aligned}$$

Use this to construct the matrix exponential e^{At}. Can you sketch some of the phase curves in 3D.