Math 231, Introduction to Differential Equations, Fall 2011
Queen's University, Department of Mathematics
email: offind@mast.queensu.ca
Office hours: Tues 10:30 11:30 or by appointment
Textbook: Elementary Differential Equations and Boundary value problems, Boyce and DiPrima, 9^{th} Edition
Grading: 15 homework, 25 midterm, 60 final
Sections in Textbook (with description) which will be covered during semester.

Introduction Classification of equations, linear and nonlinear, higher order equations, systems of first order differential equations. Applications of differential equations (preliminary discussion). Initial value problems, and solutions of differential equations. Elementary methods of solution for exponential growth and decay. **Section 2.1** linear equations (example of mass falling under constant gravitational field, with air resistance see also section 1.2) Integrating factors; **Section 2.2** separable equations (integrating factor is always separable), homogeneous solutions, methods of substitution; **Section 2.3** mixing problems and Newtonian mechanics (variable acceleration) ; **Section 2.4** Existence-uniqueness statements for linear and nonlinear equations, intervals of existence for solutions to initial value problems, determination of this interval for implicit solutions using vertical tangencies. **Section 2.5** Autonomous equations, population dynamics. Equilibria and stability. Asmyptotic properties of non equilibrium solutions. Equations dependent on a parameter, and bifurcation of equilibria. Bifurcation diagrams for equilibria as function of the parameter. Bifurcation value of the parameter. **Section 2.6** Exact equations, integrating factors. Integrating factors dependent on a single variable. Conservation laws for first order systems of equations.

Section 3.1 Homogeneous and nonhomogeneous linear equations. Linear differential operators. Homogeneous second order with constant coefficients (basic exponential solutions). Polynomial operators. Characteristic equation. Operator annihilators for exponential functions. Exponential shift for polynomial operators. Repeated roots . Applications to simple mechanical systems; **Section 3.2**Homogeneous equations. Superposition of solutions. Fundamental solutions, Wronskian test for fundamental sets of solutions ; existence uniqueness for second order linear, intervals of existence, Abels Theorem. **Section 3.3** Complex roots of characteristic equation, Eulers formula for complex exponential, graphing solutions using phase amplitude form; **Section 3.4**repeated roots of higher order operators; **Section 3.5**Nonhomogeneous equations, method of undetermined coefficients; **Section 3.6**Variation of parameters, general nonhomogeneous second order equations; Variation of parameters for higher order linear equations **Section 3.7** mechanical oscillations, simple harmonic motion, frequency and amplitude, dampened springs, critical damping, underdampened and overdamened systems. Parameter identification for dampened spring mass systems. Amplitude phase form of the homogeneous solution.

Section 4.1 General theory of nth order equations, existence uniqueness, initial value problem, interval of existence, linear independence and the Wronskian. **Section 4.2** Homogeneous equations with constant coefficients, characteristic equation and fundamental sets of solutions. Factoring higher order polynomial differential operators.

Section 7.1 Systems of first order equations. Applications (not on exam).; **Section 7.2** Matrices review ; **Section 7.3** Eigenvectors and eigenvalues review: null vectors and annihilating subspaces. **Section 7.4** Theory of first order linear systems, linear independent solutions, Wronskian and linear independence, Abel's theorem, linear combinations of fundamental sets of solutions; **Section 7.5** Eigensolutions for systems of n first order linear differential equations,

invariant linear subspaces, direction fields on these invariant spaces, phase portrait for stable and unstable node;**Section 7.6**Complex eigenvalues, rotations, phase portrait for stable and unstable focus; **Section 7.7** Fundamental matrix solutions, matrix exponential; **Section 7.8** Repeated eigenvalues, generalized eigenvectors, phase portrait for improper node.

Section 9.1 Global asymptotic stability for autonomous linear systems. The phase plane, linear systems, sketching phase portrait for planar systems; **Section 9.2** Autonomous systems equilibrium points and stability, stability diagram (page 495 8th edition, 497 9th edition). Properties of systems helpful to determine phase portrait, direction fields, nullclines, and conservation laws. **Section 9.3** Locally linear systems and introduction to competing populations. Sketching global phase portraits using local information at the equilibrium. Basins of attraction for attracting equilibria. Saddles and separtices.