Math 237, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Homework 2, Due Thursday October 6

1. Consider the differential equation and intitial value

$$6xydx + \left(3x^2 + 4\cos(y)\sin(y)\right)dy = 0, \quad y(1) = \frac{\pi}{4}$$

a) Does this initial value problem have a unique solution?

b) Find the general solution, and the solution of the initial value problem.

2. Consider the differential equation and intitial value

$$y' = 2ty^2, \quad y(a) = b$$

a) What is the largest rectangle in the $t - y$ plane for which the conclusions of the existence-uniqueness theorem are applicable.

b) Find the general solution, then compute the solution of the intial value problem $y(0) = b$ keeping the initial values as a parameter in the solution.

c) When $a = 0$, and $b > 0$,what is the interval of existence for $y(t, a, b)$? What happens to the solution as t approaches the endpoints of its interval of existence. Is this consistent with the conclusions of the existence uniqueness theorem?

3. Consider the differential equation

$$\left(3yx^2 + 2xy + y^3\right)dx + \left(x^2 + y^2\right)dy = 0$$

a) Show that there is an integrating factor of the form $u = u(x)$, and find this function which makes the equation exact.

b) Using the integrating factor found in part a), integrate the equation to find the general (implicit) solution.

c) Can you find any or all of the critical points of the function $F(x, y)$ you found in part b). Sketch some of the level curves for the function you found in part b)