

Math 237, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Homework 3, Due Thursday, October 13

1 a) Find the general solution on the interval $-\infty < t < +\infty$

$$y'' - 2y' - 3y = 0$$

b) Find the general solution

$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$$

c) Find two linearly independent solutions (show that the Wronskian is not 0 for all t)

$$(D^2 + 2D + 1)[y] = 0, \quad D = \frac{d}{dt}$$

2. Find the solution of the initial value problem, and indicate on what interval it is valid.

$$y'' + 2y' - 5y = 0, \quad y(0) = 1, \quad y'(0) = -1$$

3. Using the exponential shift find three linearly independent solutions of

$$(D^3 - 2D^2 + D)[y] = 0, \quad D = \frac{d}{dx}$$

4. Consider the third order linear homogeneous differential equation

$$(D^3 + 4D^2 + D - 6)[y] = 0.$$

a) Show that $y = e^t$ is a solution to this equation. Use this to factor the characteristic equation and find two additional exponential solutions.

b) Use the solutions you have found in part a) to find a unique solution for the initial value problem

$$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 1$$