

Math 237, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Homework 5,

Due November 10

1 a) A $\frac{1}{2}$ slug mass is attached to a vertical spring so that the spring is stretched 2 ft from its natural length. The acceleration due to gravity is 32 ft per second. The mass is started from motion with no initial velocity by displacing it $\frac{1}{2}$ ft in the upward direction. Find the subsequent motion if the mass if the medium offers a resistance of 4lbs when the velocity of the mass is 1 ft/sec. Does the system oscillate about the equilibrium configuration? Write the solution in amplitude phase form if there are oscillations. **1b)** If an external force of $\cos(2t)$ is applied to the mass, and the mass is started from equilibrium with no velocity, find the resulting differential equation, and the consequent motion of the mass.

2. Find a suitable form with fewest number of terms, for the the particular solution using the method of undetermined coefficients, but do not evaluate the constants

$$y''' - 2y'' + y' = t^3 + 2e^t$$

3. Find the general solution for $t > 0$ using variation of parameters (look for fundamental set of homogeneous solutions t^r).

$$t^2 y'' + 7ty' + 5y = t$$

4. Find the general solution and describe the behaviour of the solution as $t \rightarrow \infty$. Also sketch some representative solution trajectories in the plane either with maple, or using eigensolutions.

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix} \mathbf{x}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$