

Math 237, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Homework 6, Due Thursday, November 24

- 1) Find the general solution and describe the behaviour of the solution as $t \rightarrow \pm\infty$.

$$x' = \begin{pmatrix} 0 & 1 \\ -4 & -4 \end{pmatrix} x$$

Show that the origin in the plane is globally asymptotically stable. Also sketch the phase plane for solution trajectories in the plane. Find a spring mass problem which is equivalent to this matrix differential equation.

- 2) Find the general solution and describe the behaviour of the solution as $t \rightarrow \infty$.

Show that there is a two dimensional invariant plane of solutions which are expanding and a one dimensional invariant line of solutions which are contracting as $t \rightarrow \infty$. Sketch some representative solution trajectories in three dimensional space.

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

3) Find the matrix exponential e^{At} . Hint: use the block diagonal structure of A .

$$x' = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{pmatrix} x$$

4) Find a fundamental set of generalized eigensolutions for the matrix differential equation

$$x' = \begin{pmatrix} 2 & -1 & -4 \\ 0 & 2 & -4 \\ 0 & 1 & -2 \end{pmatrix} x$$