

Math 237, Introduction to Differential Equations, Fall 2011

Queen's University, Department of Mathematics

Homework 1, Due Thursday September 29

1. Find the solution of the following initial value problem

$$\frac{dy}{dt} + y \cos(t) = 4 \cos(t), \quad y(\pi) = -1.$$

2. Consider the following initial value problem

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}, \quad y(1) = -1.$$

a). Show that this equation is homogeneous (see page 49 Boyce and DiPrima, 8th or 9th edition).

b). Substituting the homogeneous variable $v = \frac{y}{x}$, calculate $\frac{dv}{dx}$, and find the general solution of the new differential equation in the $x-v$ plane. Graph some of the solution curves in the variables x, v , indicating carefully where vertical tangencies might occur.

c). Find the unique solution (in the x, y variables) to the initial value problem given at the beginning.

3 . A wise young engineering student with no initial capital invests k dollars per year in an investment account which gives an annual rate of return r percent. Assume that investments are made continuously and that the return is compounded continuously.

a) By modelling with a first order differential equation (explain this!) , determine the sum $S(t)$ accumulated at any time t .

- b)** If $r = 7.5$ percent, determine k so that one million dollars will be available at retirement in 40 years.
- c)** If $k = 2000$ dollars/year, determine the return rate r which must be obtained to have one million dollars available in 40 years.