MATH 237 **Fall 2010**

Homework Assignment 7

Problem 1

Show that the Laplace transform of $\cos(\alpha t)$ satisfies

$$L\{\cos \alpha t\}(s) = \frac{s}{s^2 + \alpha^2}$$

Problem 2

Use the Laplace transform method to obtain the solution to the following differential equation:

$$y'' - 2y' + 4y = 0, \qquad y(0) = 2, \quad y'(0) = 1.$$

Problem 3

Suppose that two functions g and f are related by:

$$g(t) = \int_0^t f(\tau)d\tau.$$

If $G(s)$ and $F(s)$ are the Laplace transforms of $g(t)$ and $f(t)$, show that

$$G(s) = F(s)/s$$

Problem 4

Suppose that

$$F(s) = \mathcal{L}\{f(t)\}(s),$$

exists for $s > 0$.

a) Show that if $c > 0$ then, for $s > 0$,

$$\mathcal{L}\{f(ct)\} = \frac{1}{c}F(\frac{s}{c})$$

b) Show that if $k > 0$ then, for $s > 0$,

$$\mathcal{L}^{-1}\{F(ks)\} = \frac{1}{k}f(\frac{t}{k})$$

b) Show that if $a, b > 0$ are constants, then, for $s > 0$,

$$\mathcal{L}^{-1}\{F(as + b)\} = \frac{1}{a}e^{-bt/a}f(\frac{t}{a})$$

Problem 5

Compute the inverse Laplace transform of

$$\frac{s^2 + 9s + 2}{(s-1)^2(s+3)},$$

where the inverse is a continuous function.

Hint: Use partial fraction expansion and the properties of the derivative of a Laplace transform.

Problem 6

The transfer function, $H(s)$, of a linear system is defined as the ratio of the Laplace transform of the output $y(t)$ to the Laplace transform of the input function $g(t)$, under the assumption that all the initial conditions are set to zero. If the linear system is governed by the differential equation:

$$ay'' + by' + cy = g(t),$$

verify that the transfer function is given by:

$$H(s) = \frac{1}{as^2 + bs + c}$$

Now suppose the Laplace transform of $g(t)$ satisfies $G(s) = 1$. In this case, you can see that $y(t)$ is the inverse Laplace of the transfer function, which we denote by $h(t)$. But, $G(s) = 1$ is the Laplace transform of the impulse. This is why $h(t)$ is called the impulse response of the system.