**MATH 237** **Fall 2010**

# Maple Assignment 2

### Problem 1

Consider the separable differential equation:

$$(1 + x)(1 + y)dx - dy = 0$$

First obtain the solution analytically using the appropriate integrating factor.

Using the diff and the dfieldplot commands, plot the direction field of the solutions to the equation

in the interval, $x = -5..5$, and $y = -4..4$

Comment on what happens when $y = -1$

### Problem 2

In this problem, we will use help from Maple to study a problem we considered in Assignment 1: Consider the family of parabolas defined by

$$y = kx^2,$$

where $k$ is an arbitrary constant. Find the family of curves which intersects the given family of parabolas orthogonally at each point. Hence, the directional field of the solution that we want to find

$$\frac{dy}{dx}$$

should be orthogonal to the curve

$$2kx = 2(y/x^2)x = 2y/x,$$

at every $(x, y)$ pair.

Using the diff and the dfieldplot commands, plot the direction field of the solutions to the differential equation in the interval, $x = -5..5$, and $y = -4..4$

What do the solutions look like?

*Optional: Solve the above problem by replacing $y = kx^2$ with $y = kx$: In this case, the solutions we expect to obtain will be perpendicular to a family of lines, and as such, we expect the solutions to be* circles. *Can you verify this with Maple or by derivation?*

### Problem 3

Obtain the solution to the following first analytically:

$$\frac{dy}{dt} - ty = t$$

Using the diff and the dfieldplot commands, plot the direction field of the solutions to the equation in the interval, $t = -5..5$, and $y = -4..4$

**Problem 4**

In class, we discussed a price model for supply and demand. This is given by

$$\frac{dp}{dt} = k(D(p,t) - S(p,t))$$

where, $p(t)$ is the price of a commodity, $k$ is a constant, $D(p,t)$ is the demand and $S(p,t)$ is the supply for the commodity under consideration.

Suppose $D(p,t) = A_1 - A_2 p$ and $S(p,t) = C_1 - C_2 \sin(\theta t)$

where $A_1, A_2, C_1, C_2, \theta$ are constants.

Let $A_1 = 10, A_2 = 1$, $C_1 = 5$, $C_2 = 10, k = 1$ and $\theta = 0.4$.

Let $p(0) = 10$ units of currency. Plot the solution to the differential equation in the interval, $t = 0..20$.

With the solution, interpret the change in the price as a function of the change in the supply.

**Problem 5**

During the few coming weeks, we will be using Linear Algebra occasionally. First type:

with(linalg);

To see the command list, you could type in:

?linalg;

In the following, you will perform some basic matrix algebra. Define two matrices:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -5 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 1 & -5 \end{bmatrix}$$

Compute

1. $A + B$ and $AB$
2. The inverse of $A$
3. The rank of A
4. To solve the linear equation $Ax = b$, with

   $$b = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix},$$

   compute $A^{-1}b$.