Maple Assignment 3

Problem 1

Ask maple to solve the following second order differential equation:

$$y^{(2)} + 2y^{(1)} + 5y = 0,$$

by typing in

$$> \text{Deq} := \text{diff}(\text{y}(\text{t}), \text{t}, \text{t}) + 2 * \text{diff}(\text{y}(\text{t}), \text{t}) + 5 * \text{y}(\text{t}) = 0;$$

and

$$> \text{sol1} := \text{rhs}(\text{dsolve}(\text{Deq}, \text{y}(\text{t})));$$

Note that Maple uses $\_C1$ and $\_C2$ to represent arbitrary constants. We now add initial conditions to the above problem: Let $y(0) = 2$ and $y^{(1)}(0) = -2$. We then ask Maple to execute:

$$> \text{solInitial} := \text{rhs}(\text{dsolve}(\{\text{Deq}, \text{y}(0) = 2, \text{D}(\text{y})(0) = -2\}, \text{y}(\text{t})));$$

Now let's plot the solution:

$$> \text{plot}(\text{solInitial}, \text{t} = 0..25, \text{labels} = [\text{"t"}, \text{"y"}]);$$

Problem 2

Use Maple to solve the homogenous differential equation

$$y^{(2)} - y = 0,$$

with the initial condition $y(0) = 1, \quad y^{(1)}(0) = 1$. This question was from Homework Assignment 3. Verify that Maple's solution is correct by solving it manually.

Plot your solution for $t \in [0, 5]$.

Problem 3

Solve the non-homogenous differential equation

$$y^{(2)} + y = \sin(t),$$

with the initial condition $y(0) = 1, \quad y^{(1)}(0) = 1$. Plot your solution for $t \in [0, 20]$. Comment on what happens to the solution as t grows.

Problem 4

Consider the following

$$y^{(2)} + y^{(1)} + y = t^2 \sin(4t),$$

with the initial conditions $y(0) = 1, \quad y^{(1)}(0) = 1$. From the solution Maple provides, explain which terms correspond to the complementary solution (that is, the solution corresponding to the homogenous equation) and the particular solution.

Plot your solution for $t \in [0, 20]$. Comment on the behavior to the solution as t grows.

Problem 5 [30 Points]

In this problem, we will study a system of differential equations, namely coupled spring-mass systems. Consider a modified model of the setup that we discussed in class where two masses are connected to two separate walls, and the two of them are connected with each other. The differential equations describing the system are as follows:

$$m_1 x_1^{(2)} = -k_1 x_1 + k_2(x_2 - x_1) - \beta_1 x_1^{(1)}$$
$$m_2 x_2^{(2)} = -k_2(x_2 - x_1) - k_3 x_2 - \beta_2 x_2^{(1)}$$

where m_i denotes the masses of the objects, k_i and β_i denote the spring constants.

a) Do the following:

- Write Maple commands to define seven constants and four different initial conditions.
- Define one of the ODEs on one line, and another one on another line (consult your previous homeworks and Maple's help functions).
- Find the numerical solution.
- Plot x_1 and x_2 on the interval [0,T].

The solution in such a problem can be carried over numerically as follows:

$$> soln := dsolve(\{DE1, DE2, IC\}, \{x_1(t), x_2(t)\}, numeric);$$

Here IC denotes initial conditions in the system, of the form:

$$IC := x_1(0) = 1, x_2(0) = 0, D(x_1)(0) = 1, D(x_2)(0) = 1;$$

The solution can be stored as follows:

$$> odeplot(soln, [[t, x_1(t)], [t, x_2(t)]], t = 0..t);$$

which you can use to generate the plot.

b)

After carrying over the above steps for a given set of constants, you will repeat the steps for at least six different cases for part c below.

In this part, you will define an appropriate procedure, as you did in Maple Assignment 1, to help you proceed to part c.

Your procedure should take all the constants that you defined (constants for the springs, the masses, initial conditions and T) and output a plot.

While calling the procedure, you might need to use the *restart* command, so that the Maple memory clears out the older assignments to each of the variables.

c)

Run your procedure at least six times. In your six different versions, consider a few special cases. For example, take $\beta_1 = \beta_2 = 0$, and both non-zero, and one of them zero. Then change the masses and the coefficients to answer the following:

Qualitatively explain what happens to the behavior of the solution as the spring constants, or the masses change. How do the oscillation frequency and decay rate change as functions of the spring coefficients and the masses? What happens if the masses are very different?

You may wish to use the Maple Manual, the Web and Maple's own help section to learn more about the commands.