

Maple Assignment 4

Read Chapters 8 and 9 in your Maple manual. Chapter 8 provides a discussion on systems of differential equations, and Chapter 9 discusses how to generate the so-called phase plots.

Problem 1

Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 5 \end{bmatrix},$$

Have Maple solve the following differential equation:

$$x' = Ax,$$

with the initial condition $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Note that the command at Maple for taking the exponential of a matrix is:

$$\text{exponential}(A, t)$$

Problem 2

In this assignment you will generate a family of plot for your solutions to a differential equation by doing the following: Let $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$. You are asked to plot $x_1(t)$ versus $x_2(t)$. Such a plot is termed a phase portrait.

Let

$$A = \begin{bmatrix} -2 & 1 \\ -1 & -2 \end{bmatrix},$$

and let $x(t)$ solve

$$x' = Ax,$$

For a sequence of different initial conditions, following the discussion on section 9.4 of the Maple manual, generate a list of plots (you could use the *seq* command). For the initial conditions, try 9 different combinations for $(x_1(0), x_2(0))$ and generate the phase portrait, in a single plot. For the initial conditions you could try for example all the pair combinations of $x_1(0) = -2, 0, 2$ and $x_2(0) = -2, 0, 2$.

Problem 3

Solve the following differential equation by first transforming the problem into a system of first-order differential equations and then solving the differential equation using the matrix exponential. Have Maple do the computations for you.

$$y^{(4)} + 2y^{(2)} + y = e^t,$$

with the initial conditions $y(0) = y'(0) = y^{(2)}(0) = y^{(3)}(0) = 1$. Plot $y(t)$.