MATH 237 **Fall 2010**

Maple Assignment 5

Problem 1

In this problem, you will have Maple take Laplace transforms (eventually to help you compute the solutions to differential equations). Consult Chapter 7 and compute the Laplace transform of

$$f_1(t) = \sin(t)$$
$$f_2(t) = e^t \sin(t)$$
$$f_3(t) = u(t-2)\sin(t),$$

where $u(.)$ is the step function (that is $u(t-a) = 1$, for $t \geq a$ and zero elsewhere).

Now, find the inverse Laplace transforms of

$$\frac{4}{s^2 - 4}$$
$$\frac{4}{(s-1)^2 - 4}$$
$$\frac{1}{s}\frac{1}{s^2 + 4}$$

Problem 2

Using Maple (consult Chapter 7), solve the following differential equation via Laplace transforms:

$$y^{(4)} + 2y^{(2)} + y = e^t,$$

with the initial conditions $y(0) = y'(0) = y^{(2)}(0) = y^{(3)}(0) = 1$. Plot $y(t)$.

For the remaining problems, consider the following differential equation:

$$y' = 1 - t + 4y$$

with $y(0) = 1$. The solution to this equation is

$$y(t) = \frac{1}{4}t - \frac{3}{16} + \frac{19}{16}e^{4t}$$

This solution provides a benchmark and lets us compare the performance of various numerical techniques in the following. (This example is also discussed in the recommended textbook on page 442.)

Problem 3

Write a Maple procedure which applies Euler's method, with a step size h of 0.01, and generates the sequence of approximate solutions in the interval [0,1]. Plot the solution and compare it with the plot of the accurate solution.

Problem 4

Write a procedure which applies the improved Euler's method, with a step size of 0.01, and generates the sequence of approximate solutions in the interval [0,1]. Plot the solution and compare it with the plot of the accurate solution.

Problem 5

We could have another approximation via a higher order Taylor series expansion:

$$\hat{y}(t_k + h) = \hat{y}(t_k) + y'(t_k)h + \frac{1}{2}y''(t_k)h^2$$

Write a procedure which applies a second order Taylor's method as follows, with a step size of 0.01, and generates the sequence of approximate solutions in the interval [0,1]. Plot the solution and compare it with the plot of the accurate solution.

Hint: Note that, here you will have to compute

$$y'' = \frac{d^2y}{dt^2} = \frac{d}{dt}\frac{dy}{dt} = \frac{d}{dt}y' = \frac{\partial y'}{\partial t} + \frac{\partial y'}{\partial y}\frac{dy}{dt} = \frac{\partial y'}{\partial t} + \frac{\partial y'}{\partial y}y'$$

Problem 6

Repeat problems 3 and 4 above with $h = 0.05$ and 0.001, and observe that a smaller h leads to a better approximation.

You can see that, a smaller h leads to a better approximation, with the additional complication of higher number of computations to reach from $t = 0$, to $t = 1$. If the value of h is too high, clearly the approximation can fail for certain equations. However, if the differential equation involves continuous functions, then there always exists a sufficiently small h, with which you can carry over the numerical approximation. It should be noted that, there are many other possibilities to further improve the approximation errors, such as the higher-order Taylor approximations.

Yet, another method for approximating solutions is the *Runge-Kutta method*. (Page 459 of the recommended provides a discussion on this, if you wish to learn more about the method).

As a side note, the approximation for systems of differential equations follow the same basic principles. One way is to linearize a system with the first order Taylor's approximation for each of the terms in the system.

* * Have a good winter break. * *