Math 237, Introduction to Differential Equations, Fall 2011
Queen's University, Department of Mathematics
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Office hours: Wed 10:30-11:30
Textbook: Elementary Differential Equations and Boundary value problems, Boyce and DiPrima, 9^{th} Edition
Midterm overview

Sections in Textbook (with description) which will be covered during semester.

Introduction Classification of equations, linear and nonlinear, higher order equations, systems of first order differential equations. Applications of differential equations (preliminary discussion). Initial value problems, and solutions of differential equations. Elementary methods of solution for exponential growth and decay. **Section 2.1** linear equations (example of mass falling under constant gravitational field, with air resistance see also section 1.2); **Section 2.2** separable equations (integrating factor is always separable), homogeneous solutions, methods of substitution; **Section 2.3** mixing problems and Newtonian mechanics (variable acceleration) ; **Section 2.4** Existence-uniqueness statements for linear and nonlinear equations, intervals of existence for solutions to initial value problems, determination of this interval for implicit solutions using vertical tangencies. **Section 2.6** Exact equations, integrating factors depending on single variable.

Section 3.1 Homogeneous second order with constant coefficients (exponential solutions and roots of the characteristic polynomial). Applications to electrical networks and mechanical systems; Linear differential operators. Homogeneous and nonhomogeneous equations **Section 3.2** Fundamental sets of solutions for homogeneous equations, Wronskian test for fundamental sets of solutions of linear equations; existence uniqueness for second order linear, intervals of existence, Abels Theorem **Section 3.3** Complex roots of characteristic equation, Eulers formula for complex exponential, graphing solutions using phase amplitude form; **Section 3.4**repeated roots; exponential shift for constant coefficient differential operators, critical damping for vibrating systems. Repeated roots in higher order linear homogeneous equations. **Section 3.5**Nonhomogeneous equations, method of undetermined coefficients; operator annihilators of exponential, sinusoidal and polynomial functions. Superposition of particular solutions.

Section 3.6 Variation of parameters for the general nonhomogeneous second order equations; Variation of parameters for higher order linear equations **Section 3.7** mechanical oscillations, simple harmonic motion, frequency and amplitude, dampened springs, critical damping, underdampened and overdamened systems. Parameter identification for dampened spring mass systems.

Section 4.1 General theory of nth order equations, existence uniqueness, initial value problem, interval of existence, linear independence and the Wronskian. **Section 4.2** Homogeneous equations with constant coefficients, characteristic equation and fundamental sets of solutions. Factoring higher order polynomial operators.

Section 7.1 Systems of first order equations. Applications to circuits and mixed tank reactors (not on exam); **Section 7.2** Matrices review ; **Section 7.3** Eigenvectors and eigenvalues review; null spaces and annihilating subspaces **Section 7.4** Theory of first order linear systems, linearly independent solutions, Wronskian and linear independence, Abel's theorem, linear combinations of fundamental sets of solutions (superposition); **Section 7.5** Eigensolutions for systems of n first order linear differential equations, invariant linear subspaces, direction fields on these invariant spaces, phase portrait for stable and unstable stable and unstable node;**Section 7.6**Complex eigenvalues, rotations, phase portrait for stable and unstable focus; **Section 7.7** Fundamental matrix solutions, matrix exponential; block diagonal matrices, and matrix exponential **Section 7.8** Repeated eigenvalues, generalized eigenvectors and eigensolutions, phase portrait for improper node. **7.9** Using the matrix exponential to construct particular solutions for the nonhomogeneous linear equation, variation of parameters.

Laplace transforms and nonlinear stability (also see the classroom notes from last year)

6.1 Laplace transforms as linear operator, and improper integrals.

6.2 Inverse Laplace transforms . Solving initial value problems for linear equations with constant coefficients. Application to the non homogeneous case. Exponential functions and shifting in the s-domain. Elementary Laplace transforms **Table 6.2.1**. This table will be condensed and reproducucded on your exam. It will serve as a look up table for Laplace transforms, and inverse transforms.

6.3 Step functions and Laplace transforms. Shifting in the time domain. **6.4** Differential equations with discontinuous right hand sides, using the Heaviside step function to create such functions.

6.5 Impulse functions and impulse response. Delta functions and their Laplace transforms. Impulsive differential equations. **Section 9.1** The phase plane, linear systems and global asmyptotic stability, Homogeneous systems with constant coefficients, global asymptotic stability and real parts of eigenvalues. Sketching phase portrait for planar systems; **Section 9.2** Autonomous systems equilibrium points and stability.