

Advanced Calculus, Fall 2016

Homework 1, due Thursday, September 22

1 A plane takes off from an airport with velocity vector  $(75, 150, 7.5)$  and maintains the same velocity. All units are kilometers per hour.

(i) How fast is the plane climbing vertically?

(ii) A 150 m tall skyscraper is located 5 km east and 10 km north of the airport. When will the airplane be directly over the skyscraper? (the x axis points east, the y axis points north)

(iii) when the airplane is directly above the skyscraper, how much vertical clearance is there.

2 Assume that a bicycle wheel of radius  $a$  rolls along a line without slipping. A reflector is attached to a spoke of the wheel at distance  $b$  from the center of the wheel. Find parametric equations describing the trajectory of the reflector. Note the curve traced by the reflector is called a curtate cycloid.

3. Let  $\vec{x}$  and  $\vec{y}$  denote two vectors in  $\mathbb{R}^n$ .

(a) Show that the vectors  $\|\vec{y}\|\vec{x} + \|\vec{x}\|\vec{y}$  and  $\|\vec{y}\|\vec{x} - \|\vec{x}\|\vec{y}$  are orthogonal.

(b) Show that the vector  $\|\vec{y}\|\vec{x} + \|\vec{x}\|\vec{y}$  bisects the angle between  $\vec{x}$  and  $\vec{y}$ .

4. (a) Sketch the curves whose equations in polar coordinates are respectively  $r = \cos(\theta)$  and  $r = \cos(2\theta)$  for  $0 \leq \theta \leq 2\pi$ .

(b) Describe in words and sketch the surface whose equation in cylindrical coordinates is  $r = \theta + 1$  for  $0 \leq \theta \leq 2\pi$ .

Note that we allow  $r < 0$  for certain values of  $\theta$ , which by convention means a reflection through the origin to give a point with polar coordinates  $r > 0, \theta + \pi$ .