Math/MTHE 280

Advanced Calculus, Fall 2016

Homework 2, due Thursday Sept 29

1. a) Show that for any real numbers a,b, we have $2|ab| \leq a^2 + b^2$.

(b) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Use part (a) to show that if $0 < \|(x,y)\| < \delta$, then $|f(x,y)| < \frac{\delta^2}{2}$. Prove that $f(x,y)$ is continuous on its domain.

2 (a) Compute the partial derivative functions $f_x : \mathbb{R}^2 \to \mathbb{R}$ and $f_y : \mathbb{R}^2 \to \mathbb{R}^2$, where $f(x,y)$ is the function whose defiinition is given in question 1. Notice, that you must use the definition of partial derivatives to calculate $f_x(0,0), f_y(0,0)$.

(b) Are the functions f_x and f_y continuous on $\mathbb{R}^2$? Is f differentiable at $(0,0)$?

(c) Calculate the second order mixed partial derivatives $f_{xy}(0,0)$ and $f_{yx}(0,0)$.

3. Let $q : \mathbb{R} \to \mathbb{R}$ be an arbitrary differentiable function. Show that $p(x,y) = q\left(\frac{x+y}{xy}\right)$ is a solution to the partial differential equation

$$x^2 \frac{\partial p}{\partial x} - y^2 \frac{\partial p}{\partial y} = 0.$$