Math/MTHE 280

Advanced Calculus, Fall 2016

Homework 4, due Friday Oct 7 (before 1pm!)

1(a) Let $\vec{F}, \vec{G} : \mathbb{R}^n \to \mathbb{R}^3$ be differentiable at $\tilde{\mathbf{a}} \in \mathbb{R}^\mathbf{n}$. show that the following product formula holds

$$[D(\vec{F} \times \vec{G})(\vec{a})]\vec{x} = [D\vec{F}(\vec{a})]\vec{x} \times \vec{G}(\vec{a}) + \vec{F}(\vec{a}) \times [D\vec{G}(\vec{a})]\vec{x}$$

Hint consider the cases when $\vec{x} = \vec{e}_1, \ldots, \vec{e}_n$

(b) Suppose that n=3 and that $\vec{F}(\vec{a}) = 2\vec{\imath} + \vec{\jmath} + 2\vec{k}$, and that $\vec{G}(\vec{a}) = \vec{\imath} + 2\vec{\jmath} + \vec{k}$.

$$D\vec{F}(\vec{a}) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \qquad D\vec{G}(\vec{a}) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

Find $D(\vec{F} \times \vec{G})(\vec{a}))(\vec{\imath} + \vec{\jmath} + \vec{k})$

2. Two surfaces which intersect at a point P are said to be *orthogonal at P* if their normals to their tangent planes are perpendicular at P. Show that the graphs in $\mathbb{R}^3$ of these two functions are orthogonal at every point of intersection.

$$z = \frac{1}{2}\left(x^2 + y^2 - 1\right), \qquad z = \frac{1}{2}\left(1 - x^2 - y^2\right)$$

3.(a) Consider the function

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Find the partial derivatives $f_x(0, 0), f_y(0, 0)$.

(b) Consider the parameterized curve in $\mathbb{R}^2$ given by $\gamma(t) = at\vec{\imath} + bt\vec{\jmath}$ for arbitrary $a, b \in \mathbb{R}$. Show (directly, without chain rule) that $f \circ \gamma$ is differentiable at $t = 0$ and calculate $D(f \circ \gamma)(0)$.

(c) Calculate $D(F \circ \gamma)(0)$ using the chain rule. How would you reconcile this with the answer you found in part b)?