Math/MTHE 280

Advanced Calculus, Fall 2016

Homework 4, due Thurs Oct 13 (before 2pm!)

1. Let $\mathbf{F}(x,y) : \mathbb{R}^2 \to \mathbb{R}^3$ be the function $\mathbf{F}(x,y) = \left(e^{x^2}, y\sin(\pi x), xy\right)$, and $\mathbf{G} : \mathbb{R}^3 \to \mathbb{R}^2$ be the function $\mathbf{G}(u,v,w) = \left(\cos(uv), w - u^2\right)$.

(a) Compute $\mathbf{F}(\mathbf{1},\mathbf{1})$ and denote this point as q in $\mathbb{R}^3$.

(b) Write out the composite function $\mathbf{G} \circ \mathbf{F}$, and compute directly $D\left(\mathbf{G} \circ \mathbf{F}\right)(1,1)$.

(c) Compute both $\mathbf{DF}(1,1)$, and $\mathbf{DG}(q)$

(d) Compute the product $\mathbf{DG}(q)\mathbf{DF}(1,1)$, and verify the chain rule in this case.

2. Let $\mathbf{F} : \mathcal{X} \subset \mathbb{R}^2 \to \mathbb{R}^2$ be the coordinate transformation $\mathbf{F}(r,\theta) = (r\cos(\theta), r\sin(\theta))$.

(a) Compute $\mathbf{F}\left(2, \frac{\pi}{3}\right)$ and $\mathbf{DF}\left(2, \frac{\pi}{3}\right)$.

Consider the composite function $\mathbf{H} = \mathbf{G} \circ \mathbf{F}$ where $\mathbf{G} : \mathbb{R}^2 \to \mathbb{R}$ is a differentiable function.

(b) What formula does the chain rule give for computing $\mathbf{DH}\left(2, \frac{\pi}{3}\right)$ in terms of $\mathbf{DG}$ and $\mathbf{DF}$?

(c) Suppose we know that $\frac{\partial \mathbf{H}}{\partial r}\left(2, \frac{\pi}{3}\right) = 2$, and $\frac{\partial \mathbf{H}}{\partial \theta}\left(2, \frac{\pi}{3}\right) = 4$. USe your answer from (b) to find $\frac{\partial \mathbf{G}}{\partial x}\left(1, \sqrt{3}\right)$, and $\frac{\partial \mathbf{G}}{\partial y}\left(1, \sqrt{3}\right)$. As we did in class, you will have to invert a matrix to make this computation.

(d) Suppose we are situated at the point $\left(1, \sqrt{3}\right)$. In what direction $\vec{v}$ should we move so that the instaneneous rate of change of $\mathbf{G}$ through $\left(1, \sqrt{3}\right)$ in direction $\vec{v}$ will be zero?

3. (a) Describe and sketch the graph of the function $z = \frac{1}{x^2+y^2}$.

(b) Show that the parameterized curve $\gamma(t) = (x(t), y(t), z(t)) = \left(e^t\cos(t), e^t\sin(t), e^{-2t}\right)$ lies on the graph from part (a), and that its velocity vector $\vec{v}$ at $t = \frac{\pi}{4}$ is tangent at that point to the graph .

(c) Describe what this curve does, and sketch it on the graph from part (a).