Mathematics 280

Advanced Calculus, Fall 2016

Homework 6, due Friday November 4, by NOON!

1(a) Set up a path integral, and compute the work done by the force $\tilde{\mathbf{F}} = \left(x^2 - y^2\right)\mathbf{i} + 2xy\mathbf{j}$ by moving a particle of mass m, along the square in the plane, bounded by the coordinate axes and lines $x = 3, y = 3$ with the counterclockwise orientation.

(b) Compute the path integral $\int_C \tilde{\mathbf{F}} \cdot \mathbf{dS}$ where $\tilde{\mathbf{F}} = \left(x^2 - y^2\right)\mathbf{i} + x\mathbf{j}$ and $\mathbf{C}$ is one circuit of the circle $x^2 + y^2 = 4$ in the counterclockwise direction.

2(a) Is there a vector field $\tilde{\mathbf{F}}$ so that $\text{Curl}(\tilde{\mathbf{F}}) = xy^2\mathbf{i} + yz^2\mathbf{j} + zx^2\mathbf{k}$? Explain.

(b) Is there a vector field $\tilde{\mathbf{F}}$ so that $\text{Curl}(\tilde{\mathbf{F}}) = 2\mathbf{i} + 1\mathbf{j} + 3\mathbf{k}$? If so, find one.

3.Consider the vector field $\vec{\mathbf{F}} : \mathbb{R} \times (0, \infty) \to \mathbb{R}^2$ given by

$$\vec{\mathbf{F}}(x, y) = \frac{x + xy^2}{y^2}\mathbf{i} - \frac{x^2 + 1}{y^3}\mathbf{j}$$

a) Determine if $\vec{\mathbf{F}}$ has the path independent property for every oriented path in the domain of $\vec{\mathbf{F}}$.

b) Find the work done by $\vec{\mathbf{F}}$ in moving a particle along the curve $y = 1 + x - x^2$ from (0,1) to (1,1).