Mathematics 280

Advanced Calculus, Fall 2016

Homework 7, due Friday November 11, BEFORE NOON

1) Here are three paths connecting the point (1,0,0) to the point (-1,0,0) in $\mathbb{R}^3$:

$\mathbf{C_1}$: the half circle $(\cos(t), \sin(t), 0), t \in [0, \pi]$

$\mathbf{C_2}$: the segment of a parabola $\left(-t, t^2 - 1, 1 - t^2\right), t \in [-1, 1]$

$\mathbf{C_3}$: the straight line $(-t, 0, 0), t \in [-1, 1]$.

a) For $\vec{\mathbf{F}} = (-y, x, z)$, compute $\int_{C_1} \vec{\mathbf{F}} \cdot ds$. $\int_{C_2} \vec{\mathbf{F}} \cdot ds$, $\int_{C_3} \vec{\mathbf{F}} \cdot ds$.

b) For $\vec{\mathbf{G}} = (e^{yz}, xze^{yz}, xye^{yz})$, compute $\int_{C_1} \vec{\mathbf{G}} \cdot ds$. $\int_{C_2} \vec{\mathbf{G}} \cdot ds$, $\int_{C_3} \vec{\mathbf{G}} \cdot ds$.

Hint: check whether vector field is conservative first!

2a) If $p(x, y)$ describes the density of pollution in $\frac{\text{milligrams}}{\text{square meter}}$ and x,y are measured in meters, give the units and practical interpretation of the quantity $\int\int_{\mathbf{R}} p(x, y) dA$.

b) Using Riemann sums with four equal subdivisions in each direction, find lower and upper bounds for the volume $\int\int_{\mathbf{R}} (1 + xy) dA$ over the rectangle $\mathbf{R} = [0, 2] \times [0, 4]$.

3(a) Compute the average value of the distance $\sqrt{x^2 + y^2}$ over the solid disc of radius a, centered at the origin.

3(b) Compute the total mass of the laminate circular plate of radius a

$$D = \left\{ (x, y) \mid x^2 + y^2 \leq a^2, \right\}$$

when the mass density function is $\rho(x, y) = e^{-(x^2+y^2)} \frac{\text{gm}}{\text{cm}^2}$.