

## Problem Set #10

Due: 23 November 2011

- Find a parametrization for the hyperboloid  $x^2 + y^2 - z^2 = 25$ ;
  - Find an expression for a unit normal to this surface.
  - Find an equation for the plane tangent to the hyperboloid at the point  $(a, b, 0)$  where  $a^2 + b^2 = 25$ .
  - Show that the lines  $t \mapsto (a - tb, b + ta, 5t)$  and  $t \mapsto (a + tb, b - ta, 5t)$  lie in the surface and in the tangent plane found in part (c).
- Let  $\vec{H}(x, y, z) := (e^{xy} + 3z + 5)\vec{i} + (e^{xy} + 5z + 3)\vec{j} + (3z + e^{xy})\vec{k}$ . Calculate the flux of  $\vec{H}$  through the square  $S$  of side length 2 with one vertex at the origin, one edge along the positive  $y$ -axis, one edge in the  $xz$ -plane with  $x > 0, z > 0$  and normal  $\vec{n} = \vec{i} - \vec{k}$ .
- The torus  $T$  can be parametrized by  $\vec{r}: [0, 2\pi] \times [0, 2\pi] \rightarrow \mathbb{R}^3$  where  $a > b > 0$  and  $\vec{r}(\theta, \phi) = (a + b \cos(\theta)) \cos(\phi)\vec{i} + (a + b \cos(\theta)) \sin(\phi)\vec{j} + b \sin(\theta)\vec{k}$ . Find the surface area of  $T$ .
  - Find the area of the ellipse  $E$  on the plane  $2x + y + z = 2$  cut out by the circular cylinder  $x^2 + y^2 = 2x$ .