# Problem Set #11

**Due: 30 November 2011**

**1.** Evaluate $\int_Q \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{S}}$ where $\vec{\boldsymbol{E}}(x, y, z) := ze^{x^2}\vec{\boldsymbol{\imath}} + 3y\vec{\boldsymbol{\jmath}} + (2 - yz^7)\vec{\boldsymbol{k}}$ and $Q$ is the union of the five "upper" faces of the unit cube $[0, 1] \times [0, 1] \times [0, 1]$ orient outward. The face $z = 0$ is *not* part of $Q$.

**2.** Let $S$ be the surface defined by $z = e^{1-x^2-y^2}$ with $z \geqslant 1$ oriented upward and let $\vec{\boldsymbol{H}}(x, y, z) := x\vec{\boldsymbol{\imath}} + y\vec{\boldsymbol{\jmath}} + (2 - 2z)\vec{\boldsymbol{k}}$. Calculate $\int_S \vec{\boldsymbol{H}} \cdot d\vec{\boldsymbol{S}}$.

**3.** **(a)** Consider a vector field $\vec{\boldsymbol{F}} \colon \mathbb{R}^3 \to \mathbb{R}^3$ such that $\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{F}}(x, y, z) = x^2 + y^2 + 3$. Let $M$ be the boundary of a bounded solid oriented outwads. Can $\int_M \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{S}}$ be negative?

**(b)** Find the flux of the vector field $\vec{\boldsymbol{G}}(x, y, z) = xy\vec{\boldsymbol{\imath}} + yz\vec{\boldsymbol{\jmath}} + zx\vec{\boldsymbol{k}}$ out of a sphere of radius 1 centered at the origin.