

Queen's University
Department of Mathematics and Statistics

MTHE/STAT 353
Midterm Examination February 13, 2015

- Total points = 30. Duration = 58 minutes.
- This is a closed book exam.
- One 8.5 by 11 inch sheet of notes, written on both sides, is permitted.
- A simple calculator is permitted.
- Write the answers in the space provided, continue on the backs of pages if needed.
- **SHOW YOUR WORK CLEARLY.** Correct answers without clear work showing how you got there will not receive full marks.
- Marks per part question are shown in brackets at the right margin.
- The last page contains formulas you may find useful. Please check this page first.

Marks: Please do not write in the space below.

Problem 1 [10]

Problem 2 [10]

Problem 3 [10]

Total: [30] _____

1. A family of Yellow-Faced (YF) gophers consisting of 2 parents and 3 children are kept in a laboratory. In addition to these a family of YF gophers with 2 parents and 4 children, a family of Big Pocket (BP) gophers with 2 parents and 5 children, and a family of BP gophers with 1 mother and 4 children are also kept in the laboratory. A sample of 4 gophers is selected at random from among all the gophers in the laboratory. What is the probability that the sample consists of one adult female, one adult male, and 2 children, with both adults of the same genus (either both YF or both BP). [10]

2. Let $X_1, \dots, X_n, Y_1, \dots, Y_n$ be independent Uniform(0,1) random variables. We form n rectangles, where the i th rectangle has adjacent sides of length X_i and Y_i , for $i = 1, \dots, n$. Let A_i be the area of the i th rectangle, $i = 1, \dots, n$, and define $A_{\max} = \max(A_1, \dots, A_n)$. Find the pdf of A_{\max} . [10]

3. Let X and Y be continuous random variables with joint density function

$$f(x, y) = \begin{cases} \frac{y^3}{2} e^{-y(x+1)} & \text{for } x > 0, y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the marginal pdf of X .

[5]

(b) Find the marginal pdf of Y and $E[Y]$.

[5]

Formulas:

- The Uniform(0,1) distribution has pdf

$$f(x) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$