Queen’s University
Department of Mathematics and Statistics

MTHE/STAT 353
Midterm Examination March 2, 2018

- Total points = 30. Duration = 60 minutes.
- This is a closed book exam.
- One 8.5 by 11 inch sheet of notes, written on both sides, is permitted.
- A simple calculator is permitted.
- Write the answers in the space provided, continue on the backs of pages if needed.
- SHOW YOUR WORK CLEARLY. Correct answers without clear work showing how you got there will not receive full marks.
- Marks per part question are shown in brackets at the right margin.
- The last page contains formulas you may find useful. Please check this page first.

Marks: Please do not write in the space below.

Problem 1 [10]

Problem 2 [10]

Problem 3 [10]

Total: [30] __________
1. Let $X_{ij}$, $i = 1, 2, 3$ and $j = 1, 2, 3$, be mutually independent Uniform(0,1) random variables. Let $X_{(2),j}$ be the sample median of $X_{1j}, X_{2j}, X_{3j}$, for $j = 1, 2, 3$. Find the probability that exactly one of $X_{(2),1}, X_{(2),2},$ and $X_{(2),3}$ is in the interval $[0, 1/3)$, exactly one is in $[1/3, 2/3)$, and exactly one is in $[2/3, 1)$. 

[10]
2. Let $X$ and $Y$ be jointly continuous random variables with joint pdf

$$f(x, y) = \begin{cases} 
3|x|y & \text{for } -1 \leq x \leq 1 \text{ and } x^2 \leq y \leq 1 \\
0 & \text{otherwise.}
\end{cases}$$

Find $f_X(x)$ and $f_Y(y)$, the marginal pdfs of $X$ and $Y$, respectively, as well as $E[X]$ and $E[Y]$. [10]
3. Suppose an urn contains 52 balls and suppose that 4 of the balls are numbered “1”, 4 are numbered “2”, . . . , 4 are numbered “13” (i.e., exactly 4 of the balls are numbered “i”, for i = 1, . . . , 13). Suppose all 52 balls are drawn at random, one at a time, without replacement, and the balls are paired up as they are drawn to form 26 pairs (i.e., the ith pair is the two balls drawn on draw 2i − 1 and draw 2i, for i = 1, . . . , 26). Find the expected number of pairs that have the same number on both balls. [10]