## Queen's University Department of Mathematics and Statistics

## MTHE/STAT 353

Midterm Examination March 2, 2018

- Total points = 30. Duration = 60 minutes.
- This is a closed book exam.
- One 8.5 by 11 inch sheet of notes, written on both sides, is permitted.
- A simple calculator is permitted.
- Write the answers in the space provided, continue on the backs of pages if needed.
- SHOW YOUR WORK CLEARLY. Correct answers without clear work showing how you got there will not receive full marks.
- Marks per part question are shown in brackets at the right margin.
- The last page contains formulas you may find useful. Please check this page first.

Marks: Please do not write in the space below.

Problem 1 [10]

Problem 2 [10]

Problem 3 [10]

Total: [30]

1. Let  $X_{ij}$ , i = 1, 2, 3 and j = 1, 2, 3, be mutually independent Uniform(0, 1) random variables. Let  $X_{(2),j}$  be the sample median of  $X_{1j}, X_{2j}, X_{3j}$ , for j = 1, 2, 3. Find the probability that exactly one of  $X_{(2),1}, X_{(2),2}$ , and  $X_{(2),3}$  is in the interval [0, 1/3), exactly one is in [1/3, 2/3), and exactly one is in [2/3, 1). [10]

**2.** Let X and Y be jointly continuous random variables with joint pdf

$$f(x,y) = \begin{cases} 3|x|y & \text{for } -1 \le x \le 1 \text{ and } x^2 \le y \le 1\\ 0 & \text{otherwise.} \end{cases}$$

Find  $f_X(x)$  and  $f_Y(y)$ , the marginal pdfs of X and Y, respectively, as well as E[X] and E[Y]. [10]

3. Suppose an urn contains 52 balls and suppose that 4 of the balls are numbered "1", 4 are numbered "2", ..., 4 are numbered "13" (i.e., exactly 4 of the balls are numbered "i", for i = 1, ..., 13). Suppose all 52 balls are drawn at random, one at a time, without replacement, and the balls are paired up as they are drawn to form 26 pairs (i.e., the *i*th pair is the two balls drawn on draw 2i - 1 and draw 2i, for i = 1, ..., 26). Find the expected number of pairs that have the same number on both balls. [10]