## Queen's University

## Department of Mathematics and Statistics <br> MTHE/STAT 353 <br> Midterm Examination March 2, 2018

- Total points $=30$. Duration $=60$ minutes.
- This is a closed book exam.
- One 8.5 by 11 inch sheet of notes, written on both sides, is permitted.
- A simple calculator is permitted.
- Write the answers in the space provided, continue on the backs of pages if needed.
- SHOW YOUR WORK CLEARLY. Correct answers without clear work showing how you got there will not receive full marks.
- Marks per part question are shown in brackets at the right margin.
- The last page contains formulas you may find useful. Please check this page first.

Marks: Please do not write in the space below.
Problem 1 [10]

Problem 2 [10]

Problem 3 [10]

Total: [30] $\qquad$

1. Let $X_{i j}, i=1,2,3$ and $j=1,2,3$, be mutually independent $\operatorname{Uniform}(0,1)$ random variables. Let $X_{(2), j}$ be the sample median of $X_{1 j}, X_{2 j}, X_{3 j}$, for $j=1,2,3$. Find the probability that exactly one of $X_{(2), 1}, X_{(2), 2}$, and $X_{(2), 3}$ is in the interval $[0,1 / 3)$, exactly one is in $[1 / 3,2 / 3)$, and exactly one is in $[2 / 3,1)$.
2. Let $X$ and $Y$ be jointly continuous random variables with joint pdf

$$
f(x, y)=\left\{\begin{array}{cl}
3|x| y & \text { for }-1 \leq x \leq 1 \text { and } x^{2} \leq y \leq 1 \\
0 & \text { otherwise }
\end{array}\right.
$$

Find $f_{X}(x)$ and $f_{Y}(y)$, the marginal pdfs of $X$ and $Y$, respectively, as well as $E[X]$ and $E[Y]$.
[10]
3. Suppose an urn contains 52 balls and suppose that 4 of the balls are numbered " 1 ", 4 are numbered " 2 ", ..., 4 are numbered " 13 " (i.e., exactly 4 of the balls are numbered " $i$ ", for $i=1, \ldots, 13$ ). Suppose all 52 balls are drawn at random, one at a time, without replacement, and the balls are paired up as they are drawn to form 26 pairs (i.e., the $i$ th pair is the two balls drawn on draw $2 i-1$ and draw $2 i$, for $i=1, \ldots, 26)$. Find the expected number of pairs that have the same number on both balls.
[10]

