MTHE/STAT 353 - MIDTERM EXAM
Thursday Feb.27, 2020

Instructions:

- There are 3 questions, each worth 10 marks, for a total of 30.

- The duration of the exam is 60 minutes.

- This is a closed book exam.

- One 8.5 by 11 inch sheet of notes, written on both sides, is permitted. A simple calculator is permitted.

- Write the answers in the space provided.

- SHOW YOUR WORK CLEARLY. Correct answers without clear work showing how you got there will not receive full marks.

1. Let $X = (X_1, X_2)^T$ be uniformly distributed on the positive quadrant intersected with the disk of radius 1 centred at the origin; i.e., X has joint pdf $f_X(x_1, x_2) = \frac{4}{\pi} I_{S_X}(x_1, x_2)$, where

$$S_X = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \geq 0, x_2 \geq 0, x_1^2 + x_2^2 \leq 1\}$$

Let $Y_1 = X_1^2$ and $Y_2 = X_2^2$. Find the joint pdf of $(Y_1, Y_2)^T$ and the marginal pdf of Y_1.

2. Suppose we have n disks, where the radius of disk i is X_i, and $X_1, \ldots, X_n$ are iid Uniform(0,1) random variables. Find the expected areas of the largest disk and the second largest disk.

3. Suppose an urn has one red ball, one blue ball, and one green ball. 10 draws are performed. On each draw the drawn ball is returned to the urn. What is the probability that in the 10 draws, at least 7 of the balls drawn were of the same colour and each of the colours was drawn at least once?