Queen's University

Department of Mathematics and Statistics

MTHE/STAT 353

Midterm Examination   February 26, 2021

- **THE SOLUTIONS ARE TO BE UPLOADED TO CROWDMARK**. You should have received an invitation from Crowdmark to submit your solutions.
- **Duration = 120 minutes + 30 minutes to prepare your solutions for upload to Crowdmark and to submit your solutions to Crowdmark.**
- Write your solutions using your own paper. A tablet such as an ipad is **not allowed**. Use a dark lead pencil or pen to write your solutions. Solutions which cannot be read because they are too faint will not be marked.
- Write your student number at the start of each solution and number each solution.
- Total points = 30. Each question is worth 10 marks. Marks per part question are shown in brackets at the right.
- This is a closed book exam.
- One 8.5 by 11 inch sheet of notes, written on both sides, is permitted.
- A simple calculator is permitted.
- SHOW YOUR WORK CLEARLY. Correct answers without clear work showing how you got there will not receive full marks.

1. Let $Y_1, \ldots, Y_n$ ($n \geq 2$) be independent and identically distributed discrete random variables with common pmf $f_Y(y) = \left(\frac{1}{2}\right)^{y+1}$ for $y = 0, 1, 2, \ldots$ and $f_Y(y) = 0$ otherwise (the mean of this distribution is 1). Let $X = \sum_{k=1}^{n-1} |Y_{k+1} - Y_k|$ ($|\cdot|$ is absolute value).

(a) Find $P(X = 0)$. [5]

(b) Find $E[X]$. [5]

2. Let $X_1, X_2.X_3$ be independent and identically distributed Exponential(λ) random variables (the Exponential(λ) distribution has pdf $f_X(x) = \lambda e^{-\lambda x}$ for $x > 0$ and $f_X(x) = 0$ for $x \leq 0$, and df $F_X(x) = 1 - e^{-\lambda x}$ for $x > 0$ and $F_X(x) = 0$ for $x \leq 0$). Find $P(X_1 + X_2 + X_3 \leq \frac{3}{2})$. (I want you to write out the appropriate 3-dimensional integral and evaluate it). [10]

3. Let $Y_1, \ldots, Y_n, Z_1, \ldots, Z_n$ be independent and identically distributed continuous random variables with common pdf f, where $f(t) = \frac{1}{t^2} e^{-1/t}$ for $t > 0$ and $f(t) = 0$ for $t \leq 0$. Let $X = \min\{\max(Y_1, Z_1), \ldots, \max(Y_n, Z_n)\}$.

(a) Find the pdf of X. [5]

(b) The median of the distribution of X is that value m satisfying $P(X \leq m) = \frac{1}{2}$. Find the median when $n = 5$. [5]