Queen's University
Department of Mathematics and Statistics

MTHE/STAT 353

Midterm Examination 2021

- **THE SOLUTIONS ARE TO BE UPLOADED TO CROWDMARK**. You should have received an invitation from Crowdmark to submit your solutions.
- **Duration = 120 minutes + 30 minutes to prepare your solutions for upload to Crowdmark and to submit your solutions to Crowdmark.**
- Write your solutions using your own paper. A tablet such as an ipad is **not allowed**. Use a dark lead pencil or pen to write your solutions. Solutions which cannot be read because they are too faint will not be marked.
- Write your student number at the start of each solution and number each solution.
- Total points = 30. Each question is worth 10 marks. Marks per part question are shown in brackets at the right.
- This is a closed book exam.
- One 8.5 by 11 inch sheet of notes, written on both sides, is permitted.
- A simple calculator is permitted.
- SHOW YOUR WORK CLEARLY. Correct answers without clear work showing how you got there will not receive full marks.

1. Let X_1, X_2, X_3 be independent and identically distributed discrete random variables with common pmf $p_X(x) = \left(\frac{1}{2}\right)^{y+1}$ for $x = 0, 1, 2, \ldots$ and $p_X(x) = 0$ otherwise (the mean of this distribution is 1 and the variance is 2). Consider the points (X_1, X_2) and (X_2, X_3) in the xy-plane.

(a) Find the probability that neither of the points (X_1, X_2) or (X_2, X_3) fall on the line $y = x$. (I want you to write out an appropriate triple sum and evaluate it). [5]

(b) Find the expected squared Euclidean distance between the points (X_1, X_2) and (X_2, X_3). [5]

2. Let $X_1, X_2.X_3$ be independent and identically distributed continuous random variables with common pdf $f_X(x) = \frac{1}{x^2}$ for $x \geq 1$ and $f_X(x) = 0$ for $x < 1$. Find $P(X_1X_2X_3 \leq e)$, where $e = \exp(1)$. (I want you to write out the appropriate 3-dimensional integral and evaluate it. Here's a fact you may find useful: $\frac{d}{dx}\frac{(\ln x)^2}{2} = \frac{\ln x}{x}$). [10]

3. The times to failure of components are independent exponentially distributed random variables with parameter λ (the Exponential(λ) distribution has pdf $f(t) = \lambda e^{-\lambda t}$ for $t > 0$ and $f(t) = 0$ for $t \leq 0$; df $F(t) = 1 - e^{-\lambda t}$ for $t > 0$ and $F(t) = 0$ for $t \leq 0$; and mean $\frac{1}{\lambda}$). Suppose system A consists of two components and fails as soon as one component failes, while system B consists of two components and fails as soon as both components fail. Suppose both systems start with brand new components. Let X denote the lifetime of system A and Y denote the lifetime of system B.

(a) Find $E[X - Y]$. [5]

(b) Find the probability that system B lasts at least twice as long as system A. [5]