Queen's University

Department of Mathematics and Statistics

STAT 464/864

Final Examination, December 21, 2021

Instructor: G. Takahara

INSTRUCTIONS:

- **ABSOLUTELY ZERO COLLABORATION IS ALLOWED ON THE EXAM.** There is to be no collaboration in any form on any question on any part of the exam, either in person or remotely. All work on the exam must be completed *on your own*. Any suspicion of collaboration will be flagged by me and in this case the default will be that you will receive 0 on the question. You can appeal later if you think I was wrong. If, upon further investigation, I still conclude that there was collaboration then I will pursue the consequences of academic dishonesty, which can ultimately mean withdrawal from the university.

- The exam has 5 questions, each worth 10 marks. STAT 464 students must do question 1-4 for a total of 40 marks and STAT 864 students must do all 5 questions for a total of 50 marks. Part marks are shown to the right.

- **The solutions are to be submitted through crowdmark.** You will receive an invitation from crowdmark shortly before the exam is emailed to you.

- For each question, write your solution using your own paper. Begin each solution at the start of a fresh page, and put your student number at the start of each solution.

- The exam is proctored over zoom. You must be clearly visible and unmuted in the zoom session for the duration of the exam, including when you upload your solutions to Crowdmark. The zoom link for the exam is

  https://queensu.zoom.us/j/99668067896?pwd=akJLWVBoZUczMG0xNzBuWG9kNitQZz09

Instructions continued on page 2.

- The exam is closed book. You may use a single 8.5 by 11 inch sheet of notes, written on both sides, and a Casio 991 calculator. No other aids are allowed.
- The exam is 3 hours in length, from 2pm to 5pm on December 21, 2021. At 5pm you must stop writing. You will then have 30 minutes to upload your solutions to Crowdmark. This must be done while you are still in the zoom session.
- "The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer."
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- SHOW YOUR WORK CLEARLY. Correct answers without clear work showing how you got there will not receive full marks. Marks per part question are shown in brackets at the right margin.

Student Number

1 In the following parts, ∇_d denotes the lag-d difference operator.

(a) Suppose $\{X_t\}$ has 2 seasonal components, one of period 12 and one of period 28. What is the smallest d such that $\{\nabla_d X_t\}$ will have no seasonal components? More generally, suppose $\{X_t\}$ has k seasonal components of periods $d_1, \ldots, d_k$. What is the smallest d such that $\{\nabla_d X_t\}$ will have no seasonal components? [5]

(b) Suppose $\{X_t\}$ has 2 seasonal components, one of period 5 and one of period 7, and also a quadratic polynomial trend. Give a causal filter with no more than three nonzero coefficients such that the output of the filter applied to $\{X_t\}$ will have no trend and no seasonal components. [5]

2 Let $\{X_t\}$ be a zero-mean stationary process, for $t \in \mathbb{Z}$, with ACF $\gamma_X(h)$. In each of the following parts, state whether the process $\{Y_t\}$ is necessarily stationary or not. If it is not necessarily stationary prove it. If it is necessarily stationary give the ACF of $\{Y_t\}$ in terms of the ACF of $\{X_t\}$: [10]
(a) $Y_t = (-1)^t X_t$; (b) $Y_t = X_{|t|}$; (c) $Y_t = X_{kt}$, where $k > 1$ is an integer; (d) $Y_t = X_{t^3}$.

3. Let $\{X_t\}$ be a stationary process. Compute $P(X_3 \mid X_2)$ and $P(X_1 \mid X_2)$ and show that the correlation between $X_3 - P(X_3 \mid X_2)$ and $X_1 - P(X_1 \mid X_2))$ is equal to the coefficient of X_1 in $P(X_3 \mid X_2, X_1)$. [10]

4. Let $\{X_t\}$ be the AR(2) process satisfying

$$X_t - \phi X_{t-1} - \phi X_{t-2} = Z_t, \tag{1}$$

where $\{Z_t\}$ is a zero-mean WN(σ^2) process. You may assume that ϕ is such that this AR(2) process is causal. By multiplying Eq.(1) by X_{t-k} and taking expectations, for $k = 1, 2, 3, 4, 5$, compute the ACF of $\{X_t\}$ at lags 1,2,3,4,5 in terms of ϕ. [10]

***5.** Let $\{X_t\}$ be the AR(2) process satisfying $X_t - \phi X_{t-1} - \phi X_{t-2} = Z_t$, where $\{Z_t\}$ is a zero-mean WN(σ^2) process. For what values of $\phi \in \mathbb{R}$ is the process $\{X_t\}$ causal? [10]