STUDENT NUMBER: ____________________

STAT 464/864 — Final Exam

Tuesday, December 21, 2021

Glen Takahara

Instructions: Total points = 40 for 464 and 50 for 864. Duration = 3 hours.

There are **5 questions**, each worth 10 marks. **STAT 864** must answer **all questions**. **STAT 464** students must answer **questions 1 to 4**. Write clearly in the space provided. If you need more room, continue to answer on the back of the **previous page**. **HAND IN** answers recorded on exam paper.

The exam is closed book. A single 8.5 by 11 inch sheet of notes, written on both sides is allowed. Also, a simple, non-programmable calculator (Casio 911) is allowed. Good luck!

PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

1	**2**	**3**	**4**	**5**	Total (464)	Total (864)
/10	/10	/10	/10	/10	/40	/50

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1. In the following parts, ∇_d denotes the lag-d difference operator.

(a) Suppose $\{X_t\}$ has 2 seasonal components, one of period 12 and one of period 28. What is the smallest d such that $\{\nabla_d X_t\}$ will have no seasonal components? More generally, suppose $\{X_t\}$ has k seasonal components of periods $d_1, \ldots, d_k$. What is the smallest d such that $\{\nabla_d X_t\}$ will have no seasonal components? [5]

(b) Suppose $\{X_t\}$ has 2 seasonal components, one of period 5 and one of period 7, and also a quadratic polynomial trend. Give a causal filter with no more than three nonzero coefficients such that the output of the filter applied to $\{X_t\}$ will have no trend and no seasonal components. [5]

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2. Let $\{X_t\}$ be a zero-mean stationary process, for $t \in \mathbb{Z}$, with ACF $\gamma_X(h)$. In each of the following parts, state whether the process $\{Y_t\}$ is necessarily stationary or not. If it is not necessarily stationary prove it. If it is necessarily stationary give the ACF of $\{Y_t\}$ in terms of the ACF of $\{X_t\}$:
(a) $Y_t = (-1)^t X_t$; (b) $Y_t = X_{|t|}$; (c) $Y_t = X_{kt}$, where $k > 1$ is an integer; (d) $Y_t = X_{t^3}$. [10]

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3. Let $\{X_t\}$ be a stationary process. Compute $P(X_3 \mid X_2)$ and $P(X_1 \mid X_2)$ and show that the correlation between $X_3 - P(X_3 \mid X_2)$ and $X_1 - P(X_1 \mid X_2))$ is equal to the coefficient of X_1 in $P(X_3 \mid X_2, X_1)$. [10]

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4. Let $\{X_t\}$ be the AR(2) process satisfying

$$X_t - \phi X_{t-1} - \phi X_{t-2} = Z_t, \tag{1}$$

where $\{Z_t\}$ is a zero-mean WN(σ^2) process. You may assume that ϕ is such that This AR(2) process is causal. By multiplying Eq.(1) by X_{t-k} and taking expectations, for $k = 1, 2, 3, 4, 5$, compute the ACF of $\{X_t\}$ at lags 1,2,3,4,5 in terms of ϕ. [10]

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5. Let $\{X_t\}$ be the AR(2) process satisfying $X_t - \phi X_{t-1} - \phi X_{t-2} = Z_t$, where $\{Z_t\}$ is a zero-mean WN(σ^2) process. For what values of $\phi \in \mathbb{R}$ is the process $\{X_t\}$ causal? [10]

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