STAT 464/864, Discrete Time Series Fall, 2023

A time series is a collection of observations collected over time. In this course we assume that the observations are collected at regular time intervals, i.e., the time between consecutive observations is constant, denoted by Δt. Unless otherwise specified, we will take $\Delta t = 1$.

The term time series will refer to

- An infinite sequence $\{X_t\}_{t\in\mathbb{Z}}$ of random variables, $\mathbb{Z}$ is the set of integers.
- A finite sequence $\{X_t : t=1,\dots,N\}$ of random variables representing the model for our observations.
- A finite sequence $\{x_t : t=1,\dots,N\}$, where x_t is the t^{th} observed value (a number).

We will use capital letters to denote random variables and the corresponding lower case letter to denote a realization.

A classical decomposition of a time series $\{X_t\}_{t=1}^{N}$ is

$X_t = m_t + s_t + Y_t$, where

1. $\{m_t\}$ is a deterministic trend component, (e.g., a linear trend, quadratic trend)
2. $\{s_t\}$ is a deterministic seasonal component, or periodic component, or cycle. There may be more than 1 periodic component, with different periods.

③ $\{Y_t\}$ is a random component.

We usually assume that Y_t has zero expectation, $E[Y_t]=0$.
We also assume that $\sum_{t=1}^{d} s_t = 0$, where d is the period of the periodic component.

Trend Estimation

The text discusses several approaches to estimating a trend component in Sec. 1.5, including

(a) Polynomial Regression. We assume that the trend is well fit by a polynomial of order p, for some $p \geq 1$,

i.e., $m_t = a_0 + a_1 t + a_2 t^2 + \dots + a_p t^p$,

We use linear regression to estimate the unknown coefficient $a_0, a_1, \dots, a_p$. That is, $a_0, \dots, a_p$ are chosen to minimize $\sum_{t=1}^{N} (x_t - (a_0 + a_1 t + \dots + a_p t^p))^2$.

R has a general purpose function to fit a linear regression model called lm(). The itsmr package has a function called trend( ) that is much simpler to use.

Typically a polynomial trend of order 1 or 2 would be fit, but not higher order.

(b) Moving Average Smoother

The idea is to fit the trend at time t by averaging the x_t values in some neighbourhood around time t.