STAT 464/864

Midterm Practice Problems, 2022

1. Suppose $\{s_t\}$ is a seasonal component with period $d = 2$. Show that $|\nabla^2 s_t|$ has no seasonal component, where ∇ is the difference operator.

2. Let $\{s_t\}$ and $\{r_t\}$ be seasonal components with periods a and b, respectively, where a and b are distinct positive integers that do not share any prime factors. Let $X_t = s_t + r_t + Z_t$, where $\{Z_t\}$ is a zero mean WN(σ^2) process. Find d_1 and d_2 such that $\nabla_{d_1}\nabla_{d_2}X_t$ is stationary, where ∇_d is the lag d difference operator, and find the ACF of $\nabla_{d_1}\nabla_{d_2}X_t$.

3. Suppose $\{X_t\}$ and $\{Y_t\}$ are independent (i.e., X_r and Y_s are independent for every r and s), 0-mean, stationary processes with autocovariance functions $\gamma_X(h)$ and $\gamma_Y(h)$, respectively. Let $Z_t = X_t Y_t$. What is the ACVF and ACF of $\{Z_t\}$?

4. For d odd, give a symmetric linear filter (i.e., $a_{-j} = a_j$) that eliminates seasonal components with period d and passes linear trends. What if d is even?

5. Consider the exponential smoother with smoothing parameter $\alpha \in (0, 1)$ and let $\{X_t\}$ be the result of applying this smoother to $\{Z_t\}$, where $\{Z_t\}$ is a zero-mean WN(σ^2) process, i.e., $X_t = \sum_{i=0}^{\infty} \alpha(1-\alpha)^i Z_{t-i}$. Find the autocovariance function (ACVF) of $\{X_t\}$ at all lags h.

6. Let $X_t = Y_0 s_t + Y_1 s_{t-1}$, where $\{s_t\}$ is seasonal with period 2, and $Y_t = Z_t + \theta Z_{t-1}$ is an MA(1) process with MA coefficient θ, and $\{Z_t\}$ is a zero-mean WN(σ^2) process. Compute the ACF of $\{X_t\}$.

7. Show that the two MA(1) processes

$$\begin{aligned} X_t &= Z_t + \theta Z_{t-1}, \quad \{Z_t\} \sim \text{WN}(0, \sigma^2) \\ Y_t &= \tilde{Z}_t + \frac{1}{\theta}\tilde{Z}_{t-1}, \quad \{\tilde{Z}_t\} \sim \text{WN}(0, \sigma^2\theta^2), \end{aligned}$$

where $0 < |\theta| < 1$, have the same autocovariance function.