Queen's University

Department of Mathematics and Statistics

STAT 464/864

Midterm Examination March 18, 2021

- **THE SOLUTIONS ARE TO BE UPLOADED TO CROWDMARK**. You should have received an invitation from Crowdmark to submit your solutions.
- Write your solutions using your own paper or directly on the exam sheets (assuming you print out the exam). You may also use a writing tablet such as an ipad to write your solutions. If writing on paper, use a dark lead pencil or pen to write your solutions. Solutions which cannot be read because they are too faint will not be marked.
- Start each solution on a new page. Write your student number at the start of each solution and number each solution.
- **Duration = 90 minutes + 30 minutes to prepare your solutions for upload to Crowdmark and to submit your solutions to Crowdmark.**
- The exam is **open book**. This means you can use the class notes and the textbook. You may also use your computer. However, **you may not use any device for communication with another person (see the next item)**.
- **ABSOLUTELY ZERO COLLABORATION IS ALLOWED**! There is to be no collaboration in any form on any question on any part of the exam. All work on the exam must be completed *on your own*.
- The exam has 3 questions. Questions 1 and 2 are for STAT 464 students. STAT 864 students must do all 3 questions. Note that STAT 464 students *must* do questions 1 and 2. If you are in STAT 464 and do question 3 *it will not be marked*.
- Each question is worth 10 marks, for a total of 20 for STAT 464 and 30 for STAT 864. Marks per part question are shown in brackets at the right.
- SHOW YOUR WORK CLEARLY. Correct answers without clear work showing how you got there will not receive full marks.
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1. (a) Let $\{s_t\}$ and $\{r_t\}$ be seasonal components with periods a and b, respectively, where a and b are distinct positive integers that do not share any prime factors. Let $X_t = s_t r_t + Y_t$, where $Y_t = \phi Y_{t-1} + Z_t$ is an AR(1) process with AR coefficient ϕ satisfying $|\phi| < 1$ and $\{Z_t\}$ is a zero mean WN(σ^2) process. Find d such that the lag d difference operator applied to $\{X_t\}$ (i.e., $\nabla_d X_t$) is stationary, and find the ACF of $\nabla_d X_t$. [6]

(b) Let $\{s_t\}$ be a seasonal component with period d, and let a, b, and c be constants. Let $X_t = (a + bt + ct^2)s_t + Y_t$, where $\{Y_t\}$ is a zero mean stationary process. Show that $\nabla_d^3 X_t$ is stationary. [4]

2. Let $\{Z_t\}$ be a zero mean zero WN(σ^2) process. For each of the following parts, state whether the given process $\{X_t\}$ is stationary or not. If it is not stationary, prove it. If it is stationary, compute its ACF.

(a) $X_t = (-1)^t Z_0 + (-1)^{t+1} Z_1$. [3]

(b) $X_t = Z_0 s_t + Z_1 s_{t-1}$, where $\{s_t\}$ is seasonal with period 2. [4]

(c) $X_t = \sum_{j=0}^{d} \frac{1}{d+1} Z_{t-j}$, where d is a fixed positive integer. [3]

*3. Let $\{Z_t\}$ be a zero mean WN(σ^2) process, and let $X_t = Z_t + \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j-1} Z_{t-j}$. Find the ACVF and the ACF of $\{X_t\}$. [10]