Event-triggered stabilization over digital channels

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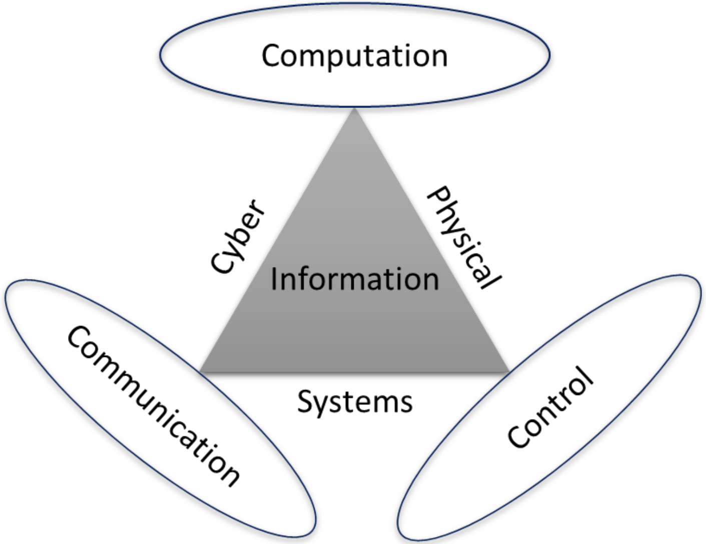
Outline

- Preliminaries
  - Cyber-physical systems
  - Data-rate theorem
- Event-triggered stabilization over digital channels
  - Scalar systems
  - Experimental Validation
  - Zeno Behavior
  - Event-triggered vs. Time-triggered
  - Vector systems
  - Exponential convergence
- Discussion and future work

PRELIMINARIES

Cyber-physical systems (CPS)

- Largely regarded as the next-generation engineering systems
- Integration of computing, communication, and control
- Arising in diverse areas such as robotics, energy, and transportation



Cloud robots and automation systems

- An example of CPS
  - An emerging field in robotics and automation
  - Cloud enables robots to use shared resources
  - Feedback loop is closed over a communication channel
    - Noisy and subject to delay

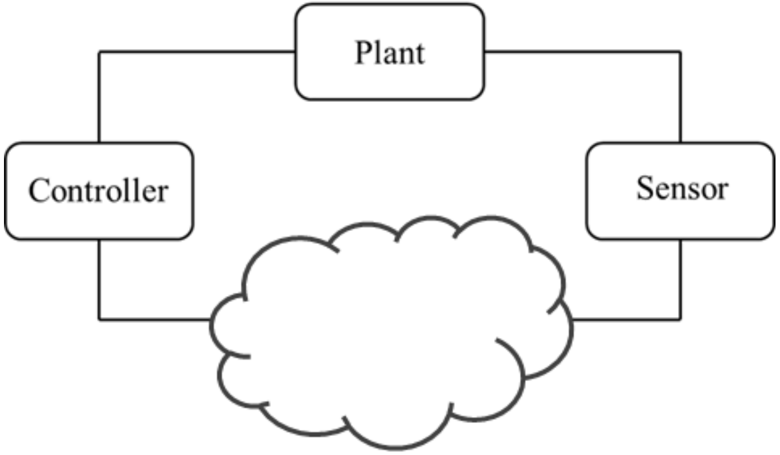
Networked Control Systems

- Plant is scalar

$$\dot{X} = aX(t) + bU(t) + W(t)$$
$$|W(t)| \leq m$$

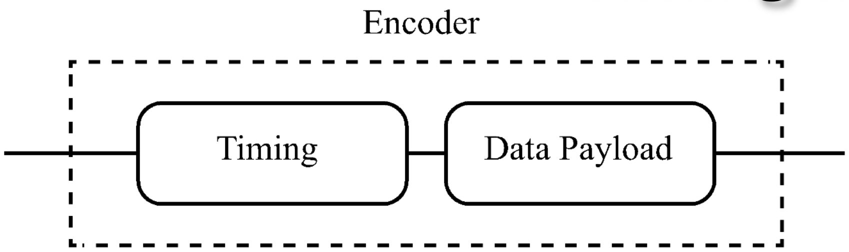
- Plant is unstable

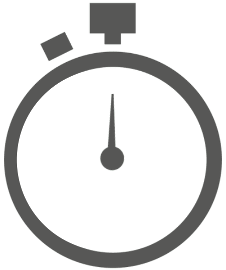
- Communication channel is subjected to a finite data rate and bounded unknown delay



Communication Channel

Timing information





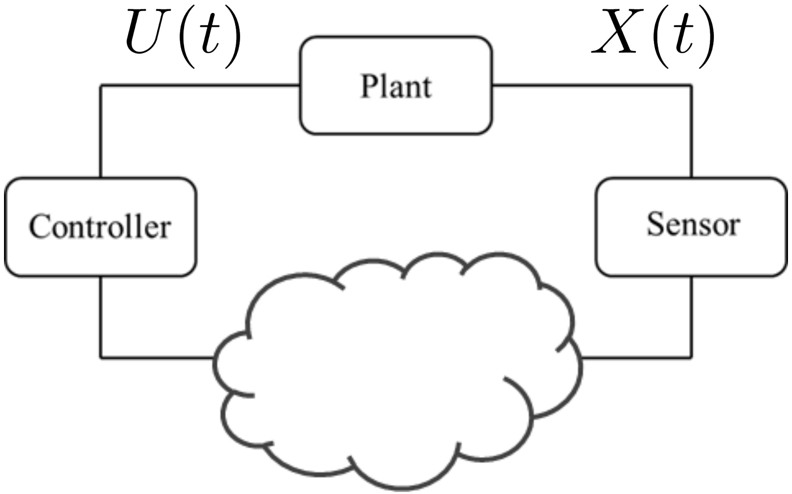
Delay

2.0 s

1.0 s

0.5 s

Timing Information
+
Data Payload



Communication Channel

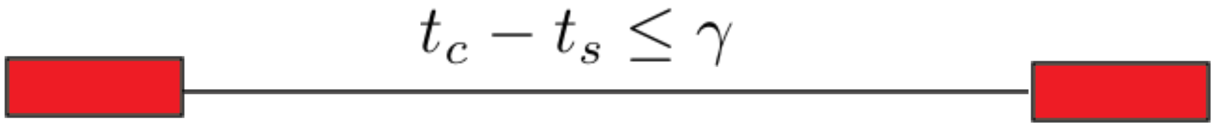
1 0 1 1

1 0 0 1

0 0 1 1

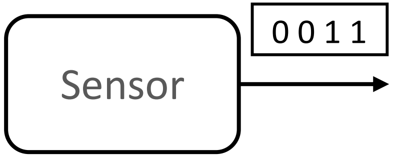
Transmission with delay

- Packet transmission time t_s
- Packet reception time t_c
- Delay $t_c - t_s \leq \gamma$

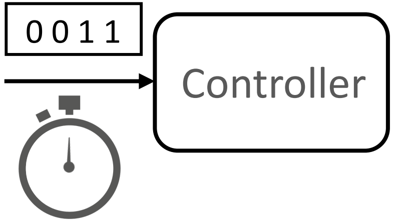


Information rate

- $b_s(t)$ number of bits in data payload transmitted up to time t
  - Information transmission rate
$$R_s = \limsup_{t\to\infty} \frac{b_s(t)}{t}$$
  - The rate at which the sensor transmits data payload



- $b_c(t)$ be the amount of information measured in bits included in data payload and timing information received at the controller until time t
  - Information access rate
$$R_c = \limsup_{t\to\infty} \frac{b_c(t)}{t}$$
  - The rate at which controller receives information

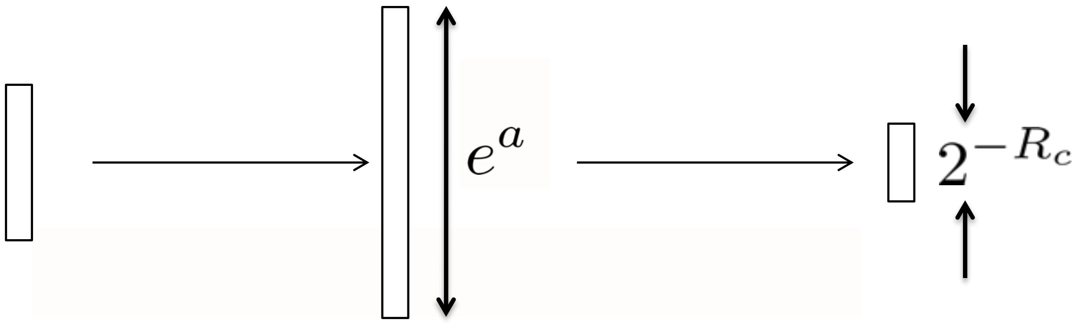


Data-rate theorem

- We can stabilize the system if and only if the information access rate

$$R_c > \frac{a}{\ln 2}$$ → entropy rate of the plant

State uncertainty



Data-rate theorem

- Balance between production and consumption of information

- This information can be supplied to the controller by data payload as well as timing

$$R_c > \frac{a}{\ln 2} \qquad\qquad R_s?$$

EVENT-TRIGGERED STABILIZATION OVER DIGITAL CHANNELS

Event-triggering review

- Periodic control is the most common and perhaps simplest solution for digital systems.

  – Step 1: Good Dog

  – Step 2: Good Dog

  – Step 3: Bad Dog

  – Step 4: Good Dog

  .

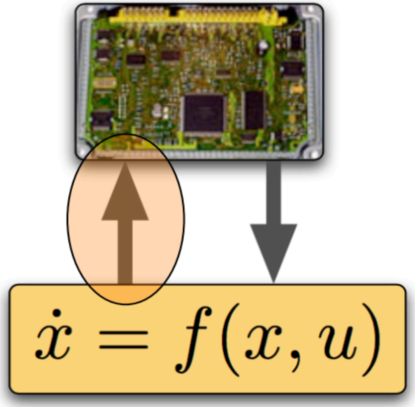
  .

  .

Genibo SD Robot Dog

Event-triggering review

- In CPS we need to use the shared resources efficiently
  - Periodic control can be inefficient
  - Event-triggered control transmit sensory data in an opportunistic manner



Event-triggering review

- The main concept of event-triggered control is to transmit sensory data only when needed
  - Step 1: --
  - Step 2: --
  - Step 3: Bad Dog
  - Step 4: --

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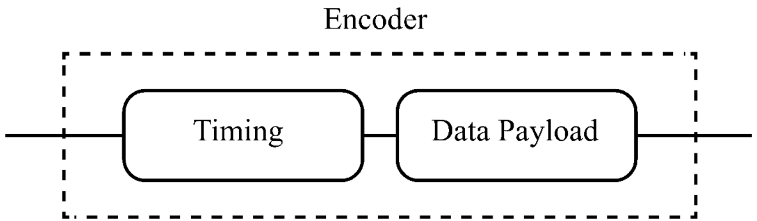
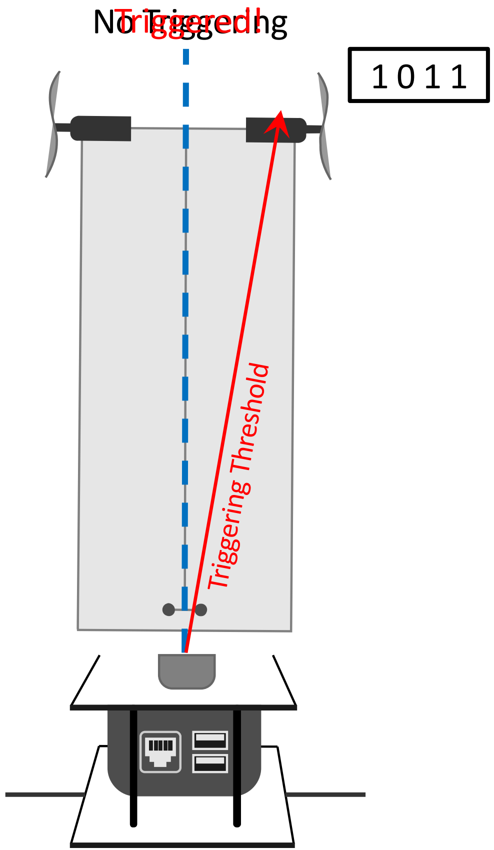
- “Wise men speak because they have something to say” — Plato

State dependent timing information encoding

- Our goal is to propose an event-triggering strategy that utilizes timing information by transmitting in a state-dependent fashion.



  - Intuitive example: stabilization of an inverted pendulum over a digital communication channel



Input-to-state practical stability (ISPS)

- Encoding-decoding scheme, which encodes information in timing via event-triggering, to achieve ISpS

$$|X(t)| \le \beta(|X(0)|, t) + \psi(|W|_t) + \chi(\gamma) + \zeta(|W|_t, \gamma).$$

$$\beta \in \mathcal{KL} \qquad \psi \in \mathcal{K}_\infty(0) \qquad \chi \in \mathcal{K}_\infty(d) \qquad \zeta \in \mathcal{K}^2_\infty(0, d')$$

$$|W|_t = \sup_{s \in [0,t]} |W(s)|$$

  - For a fixed γ, this definition reduces to the standard notion of ISpS (Z-P Jiang, A. R. Teel, L. Praly- 94 and Sharon, Liberzon- 12).

  - Given that the initial condition, delay, and system disturbances are bounded, ISpS implies that the state must be bounded at all times.

State estimation error

- Plant

$$\dot{X} = aX(t) + bU(t) + W(t)$$

- $\hat{X}(t)$ the state estimation constructed at the controller
  - Inter-triggering times

$$\dot{\hat{X}}(t) = A\hat{X}(t) + BU(t), \quad t \in (t_c^k, t_c^{k+1})$$

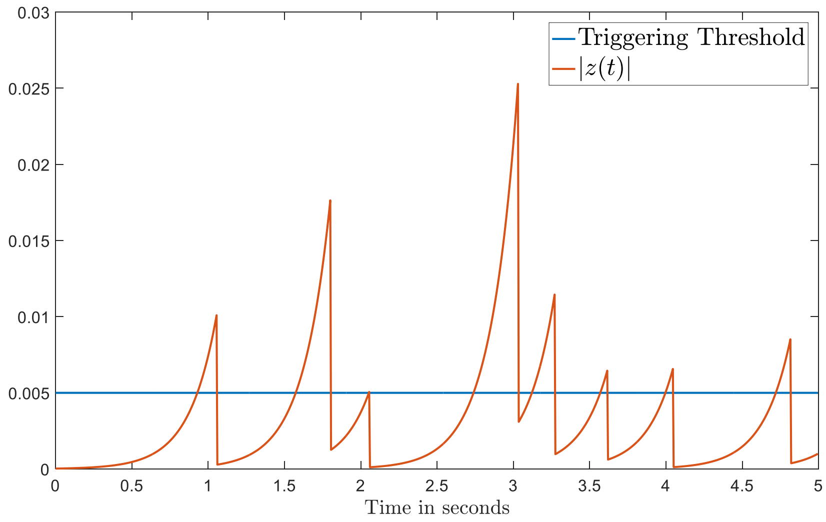
- We assume the sensor can also compute the same estimate $\hat{X}(t)$ via a feedback acknowledgment
  – Communication via control input
  – Control input is known at the sensor and it jumps only at each reception times

- State estimation error

$$Z(t) = X(t) - \hat{X}(t)$$

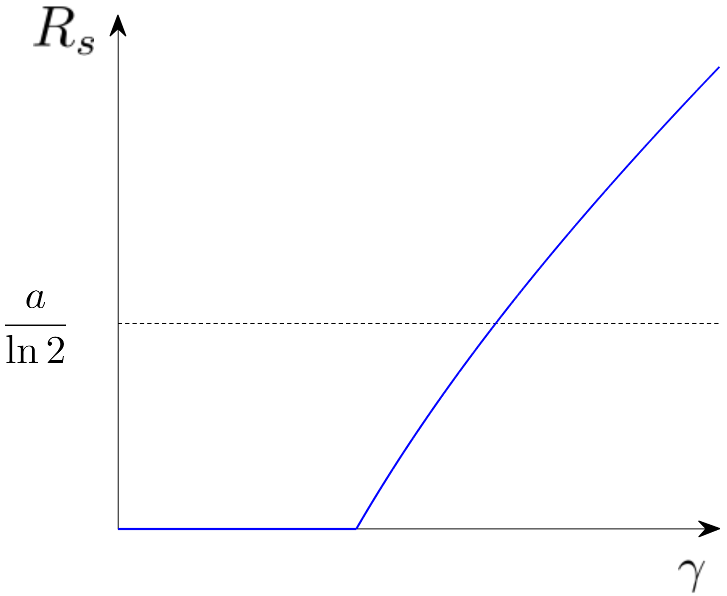
Triggering strategy

- Triggering criterion $|Z(t_s)| = J$
  - Triggering threshold J
  - $|Z(t_c^+)|$ is always below the triggering threshold
  - $|Z(t)|$ is bounded



Information transmission rate

- Required information transmission rate vs delay upper bound
  - Small values of delay
    - Timing information is substantial
    - R_s is arbitrarily close to zero
  - As delay increases
    - Timing information becomes out of date
    - R_s begin to increase
  - Large value of the delay
    - Uncertainty at the controller increases
    - State estimation error should be below the threshold at the reception time
    - R_s exceeds the rate imposed by the data-rate theorem



Kh, Hedayatpour, Cortés, Franceschetti- 21

Challenges

- Packet size
  - Necessary Condition

$$\#\text{bits} \geq \log \frac{m(\text{uncertainty set})}{m(\text{covering ball})}$$

  - Sufficient condition
    - We designed an encoding-decoding scheme
      - Encode a quantized version of the triggering time in the data payload and timing

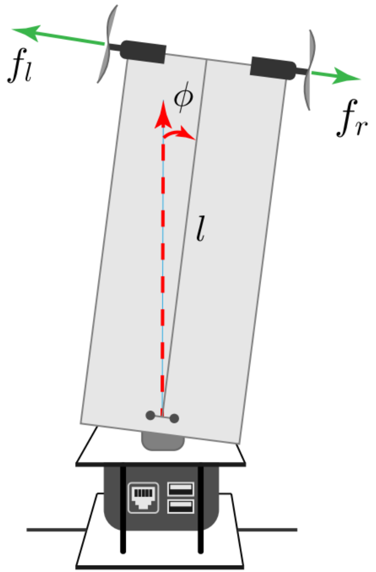
- Triggering rate

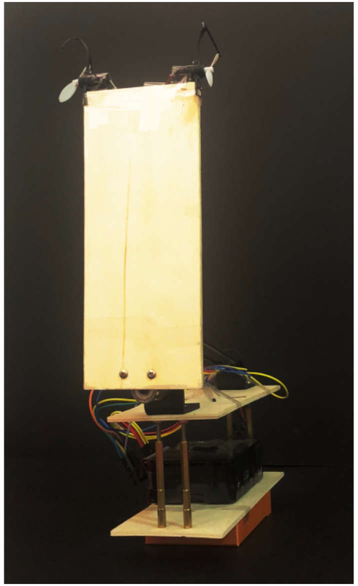
$$\text{Frequency} = \limsup_{N\to\infty} \frac{N}{\sum_{k=1}^{N} k^{th}\text{inter-event time}}$$

  - Necessary Condition: lower bound
  - Sufficient condition: upper bound

Experimental Validation

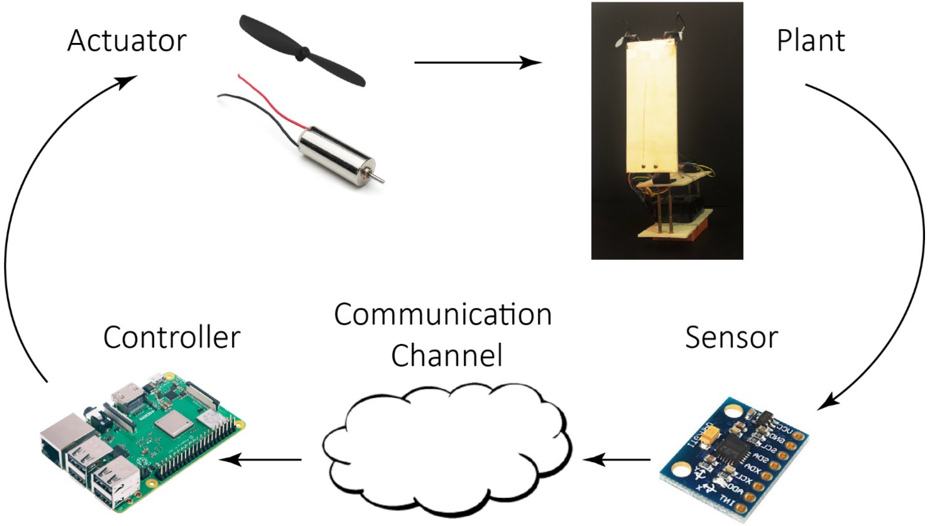
- Laboratory-scale inverted pendulum
  - Using linearized model
  - Stabilization around unstable equilibrium point





Experimental Validation

- Off-the-shelf components
  - Raspberry Pi Model 3
  - Two small DC motors
  - Two identical propellers
  - MEMS sensor
    - 3-axis accelerometer
    - 3-axis gyroscope
  - Complimentary filter



  - Details of these experiments and validation
    - Kh, Hedayatpour, Franceschetti- 19

Experiment 1

- Delay upper bound
  - 2 Sampling Times
- Packet Size
  - 1 bit
- Number of Samples
  - 6541
- Number of Triggering
  - 170

- Information Transmission Rate — Entropy Rate of the System

8.6633 bit/sec $<$ 10.5461 bit/sec

Experiment 2

- Delay upper bound
  - 3 Sampling Times
- Packet Size
  - 3 bit
- Number of Samples
  - 6333
- Number of Triggering
  - 146

- Information Transmission Rate

23.0526 bit/sec

>

Entropy Rate of the System

10.5461 bit/sec

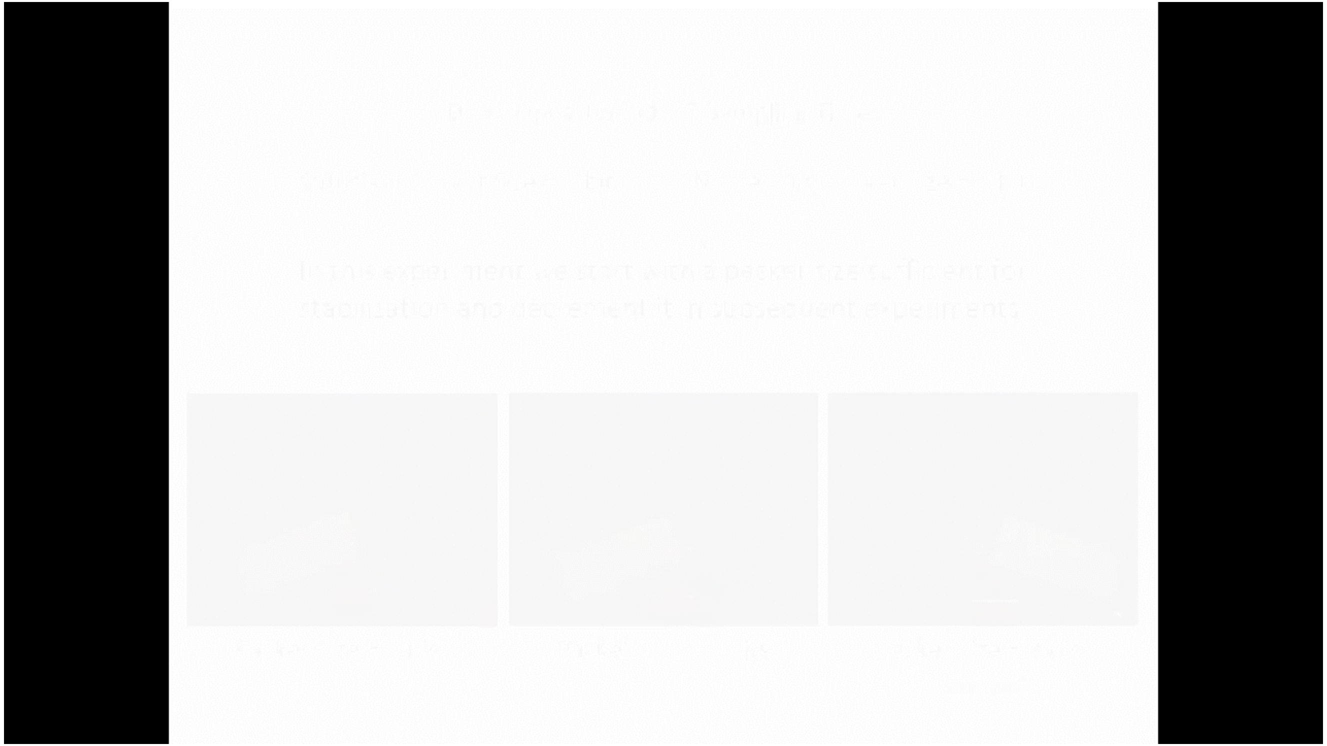
Experiment 3

- Delay upper bound
  - 7 Sampling Times

- Packet Size
  - Sufficient Packet Size:
    - 5 bit
  - Necessary Packet Size:
    - 1 bit

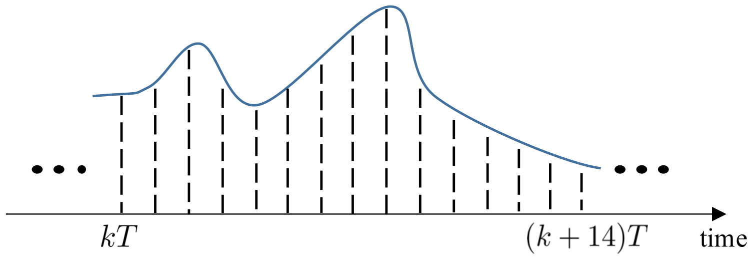
- In this experiment we start with a packet size sufficient for stabilization and decrease it in subsequent experiments

Experiment 3

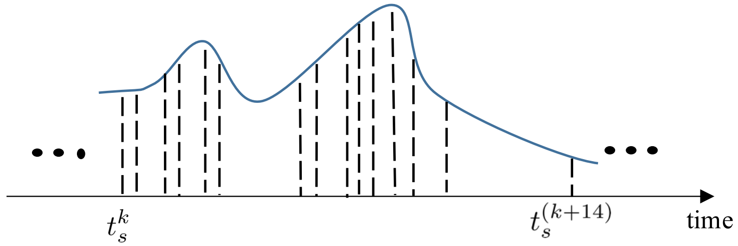


Zeno Behavior

- Periodic control
  - Equal-distance sampling

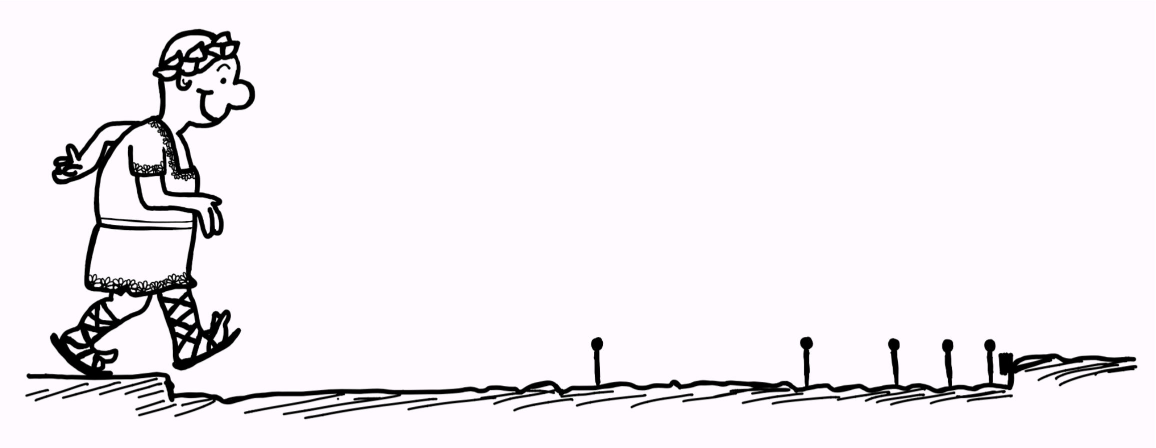


- Event-triggered control
  - Sporadic sampling
  - Hybrid phenomenon
    - Zeno behavior



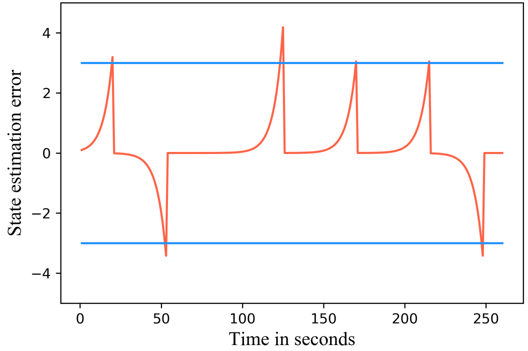
Zeno Behavior

- A paradox by ancient Greek philosopher Zeno of Elea
  - "That which is in locomotion must arrive at the half-way stage before it arrives at the goal."
  - We should never be able to reach any destination!

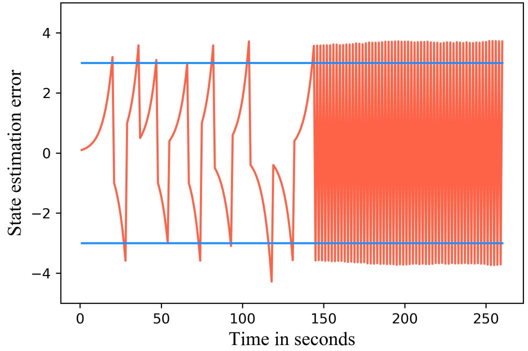


Zeno Behavior

- Normal realization



- Zeno realization
  - Degenerate behavior of some event-triggering strategies
  - Infinite number of triggering events occurring in a finite amount of time

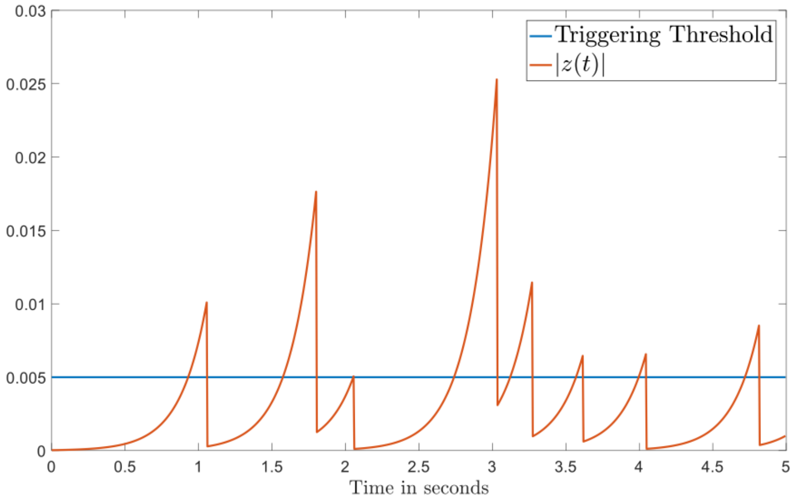


Zeno Behavior

- Event-triggering strategies
  - Guarantee stability
  - Rule out the Zeno behavior
- Design Packet size
  - For $0 < \rho_0 < 1$

$$|z(t_c^+)| \leq \rho_0 J$$

  - Uniform lower bound on the inter-triggering times



Time-triggering vs event-triggering

- We compared our results against information access rate $R_c > \frac{a}{\ln 2}$

- In a time-triggered strategy $R_s?$

  – Time-triggered strategy

$$t_s^0 = 0, \quad t_s^{k+1} = t_s^k + (\lfloor \Delta_k / T \rfloor + 1)T$$

  – Similar to our event-triggering setup a packet is transmitted only after the previous packet is received.

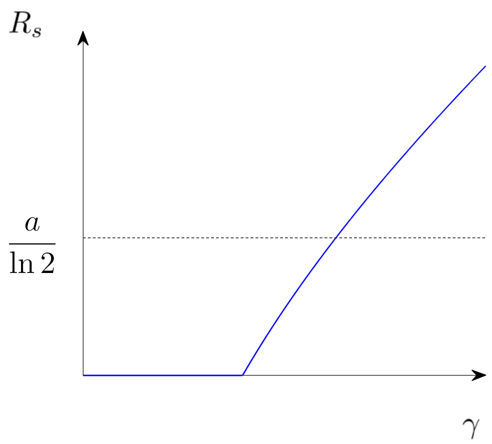
Time-triggering vs event-triggering

- Time-triggering strategies
  - Delay dependent
  - Does not exploit timing information

$$R_s \geq \frac{a(\lfloor \frac{\gamma}{T} \rfloor + 1)}{\ln 2}$$

Kh, Tallapragada, Cortés, Franceschetti- 17

- Event-triggering strategies
  - State and delay dependent
    - Transmit sensory data only when needed
    - Exploit timing information



Vector systems

- Data-rate theorem

$$R_c > \frac{Tr(A)}{\ln 2}$$

- Time-Triggering

$$R_s \geq \frac{Tr(A)(\lfloor \frac{\gamma}{T} \rfloor + 1)}{\ln 2}$$

- Event-Triggering

Vector systems

- Triggering criterion
  - Various ways $||z(t_s)||_2 = v(t_s)$

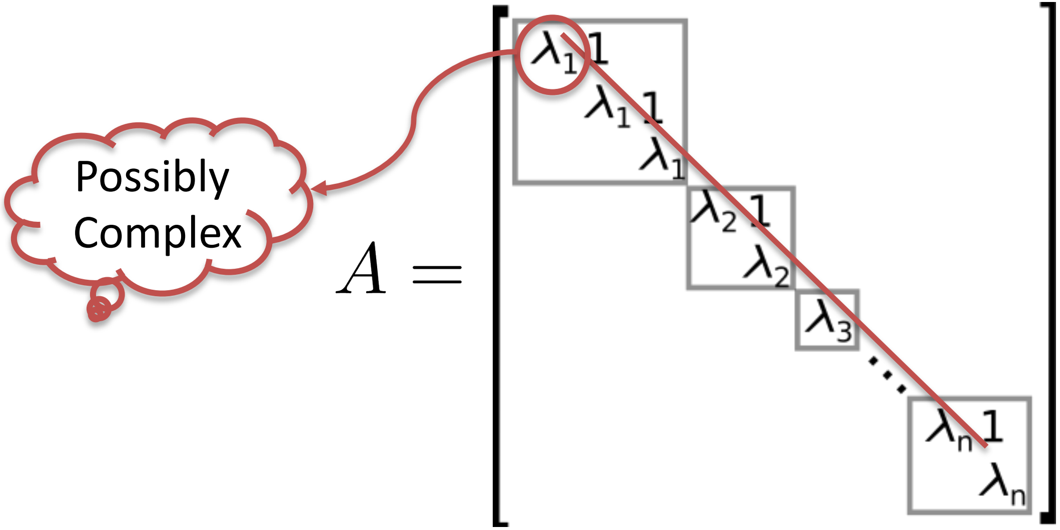
    - Coordinate by coordinate analysis $|z_i(t)| = J_i$
      - This corresponds to treating the n-dimensional system as n scalar coupled systems.

Vector Systems: Communication Channel

- We assume that there are n parallel finite-rate digital communication channels between each coordinate of the system and the controller, each subject to unknown, bounded delay

- In the case of a single communication channel, we can consider the same triggering strategy, but an additional $\lceil \log n \rceil$ bits should be appended at the beginning of each packet to identify the coordinate it belongs to

Vector systems: Jordan block



– Off-diagonal ones make coupling between states
  - Kh, Tallapragada, Cortés, Franceschetti- 20

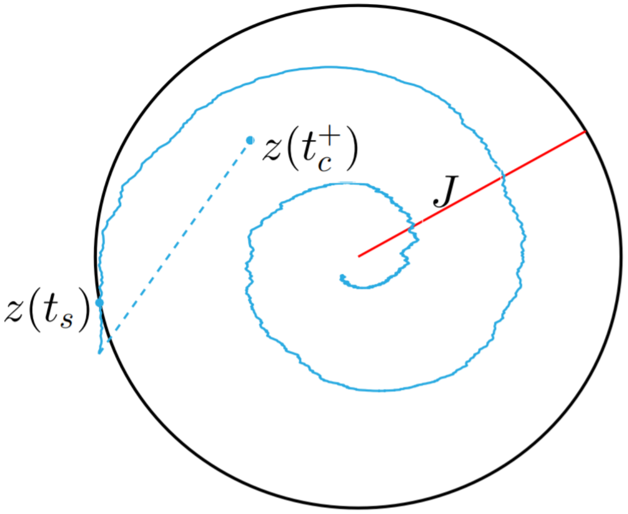
Extension to complex linear systems

- Plant $$\dot{X} = aX(t) + bU(t) + W(t)$$
  - Bounded disturbances $$\|W(t)\| \leq m$$
  - Data-rate theorem extension $$R_c > \frac{2Re(a)}{\ln 2}$$
    - This information can be supplied to the controller by data payload as well as timing

$$R_s?$$

Triggering strategy

- Triggering criterion $\|Z(t_s)\| = J$
  - Triggering radius J
  - $\|Z(t_c^+)\|$ is always inside the triggering circle
  - $\|Z(t)\|$ is bounded



The encoding

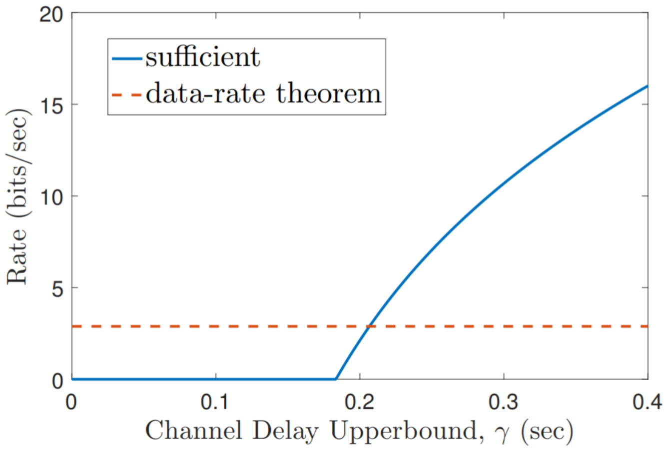
100010101010011010010	10101010010110101010100101000

A uniform quantization of the phase at which the state estimation error hits the triggering circle

A quantized version of triggering time which is constructed like our encoding process for linear scalar systems.

Information transmission rate

- Required information transmission rate for stabilization
  - Similar to scalar real plant
    - For small values of the delay, R_s is smaller than the rate required by the data-rate theorem



Kh, Hedayatpour, Cortés, Franceschetti- 21

Exponential convergence

- Exponential convergence of the estimation error or the plant state
  - $\forall t > 0 \qquad |z(t)| \leq |z(0)|\, e^{-\sigma t}$ or $\forall t > 0 \qquad |x(t)| \leq |x(0)|\, e^{-\sigma t}$

$$R_c \geq \frac{A + \sigma}{\ln 2}$$

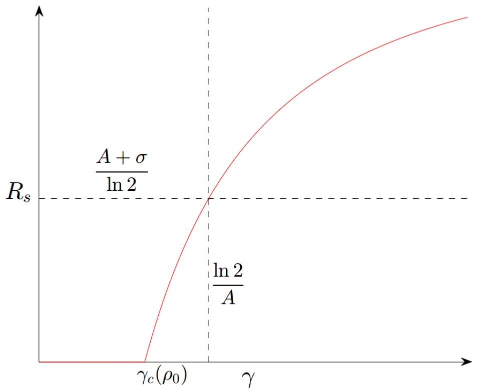
  - The access rate should be larger than entropy rate of the plant + convergence rate
    - Kh, Tallapragada, Cortés, Franceschetti- 17
    - Estimation entropy (Liberzon, Mitra -17)

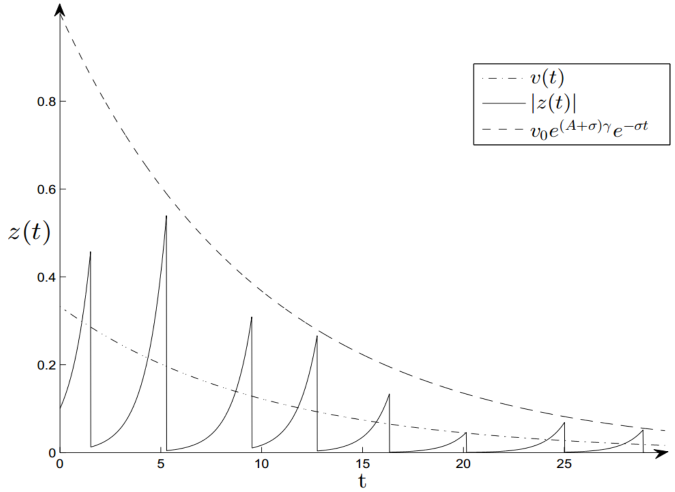
Exponential convergence

- Time-triggering

$$R_s \geq \frac{(a+\sigma)(\lfloor \frac{\gamma}{T} \rfloor + 1)}{\ln 2}$$

- Event-triggering





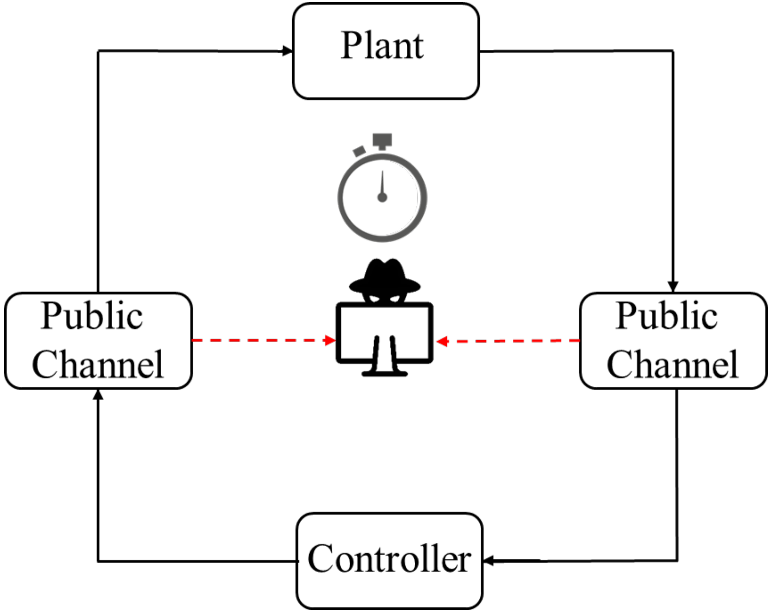
- Kh, Tallapragada, Cortés, Franceschetti- 20

DISCUSSION AND FUTURE WORK

Security and privacy issues

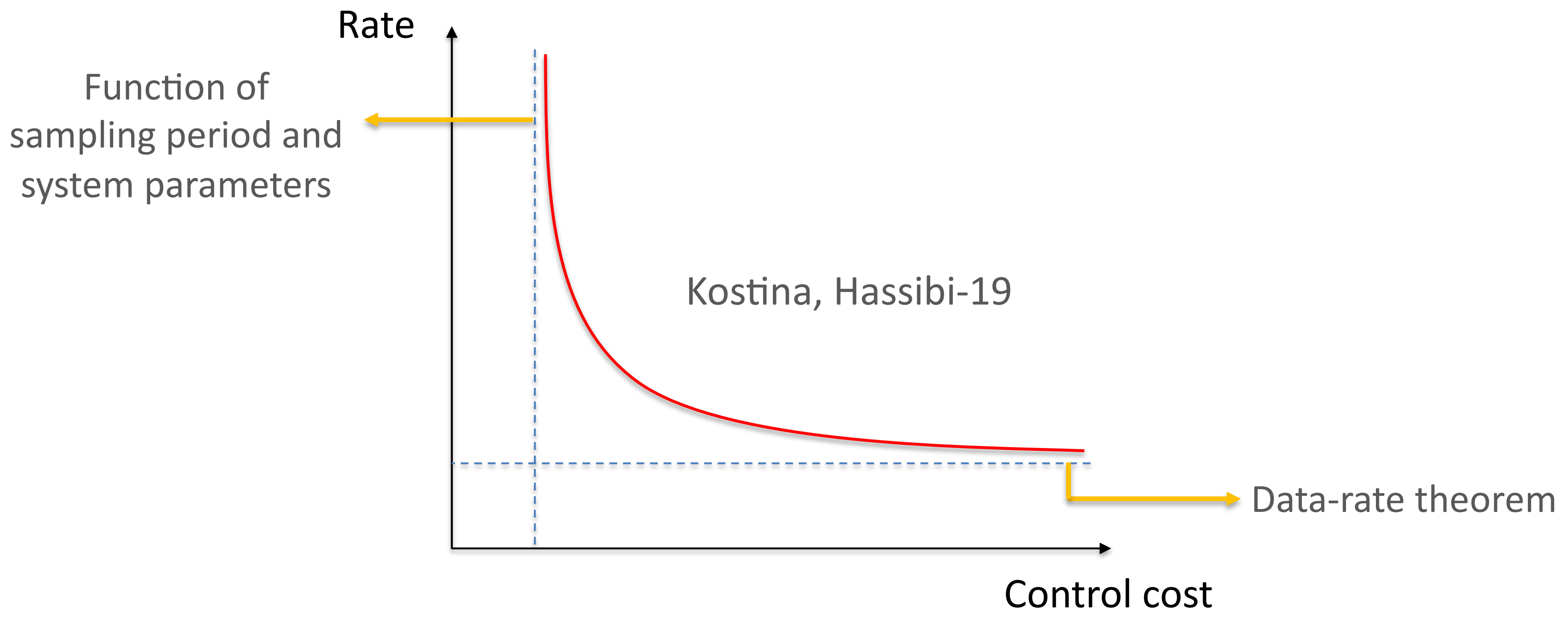
- Adversaries might take advantage of the inherent timing information in even triggering

- In context of
  - Differential privacy
    - Cortes et al, CDC 2016
  - Learning-based attacks
    - Khojasteh et al, TCNS 2021.



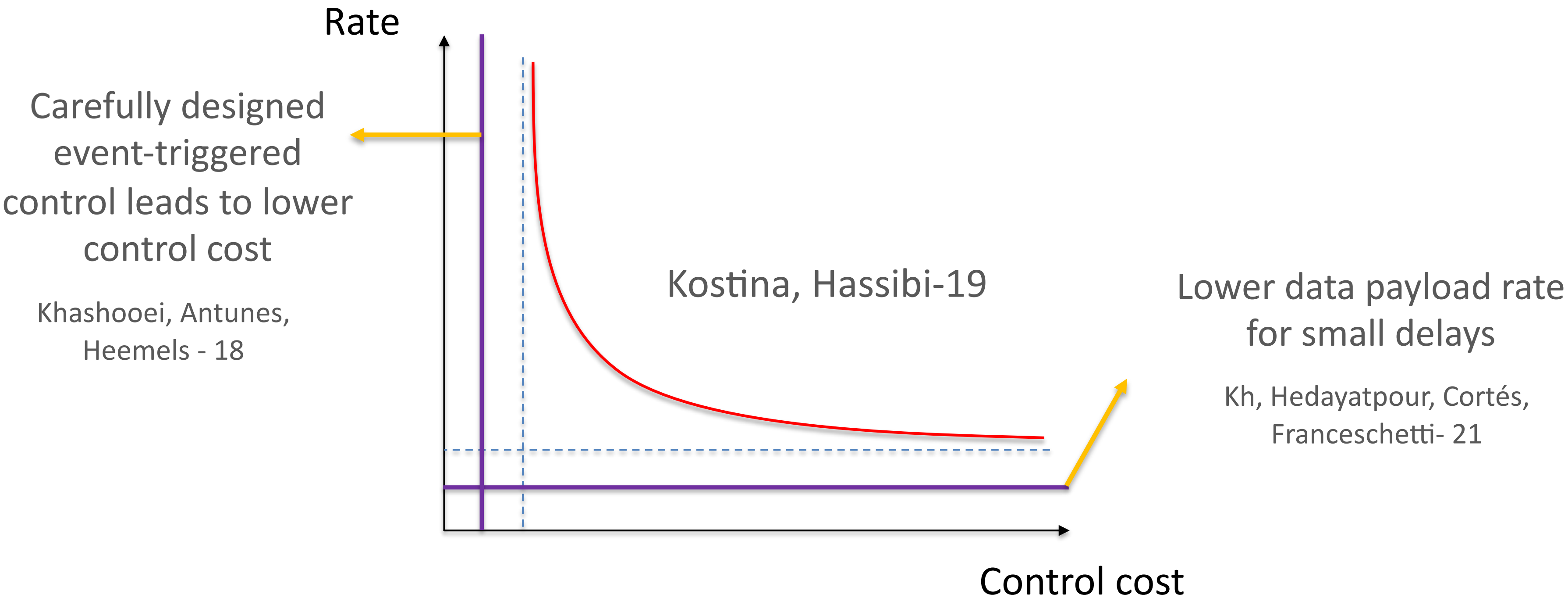
Rate-cost tradeoffs in periodic control

- Appropriate communication rate to achieve a control objective



Rate-cost tradeoffs in event-based control

- The event-triggering can improve this results in two aspects



Nonlinear Systems

- Plant

$$\dot{X} = f(X(t), U(t), W(t))$$

  - Bounded disturbances

$$|W(t)| \leq m$$

  - Locally Lipschitz

$$|f(X, U, W) - f(\hat{X}, U, 0)| \leq L_x|X - \hat{X}| + L_w|W|$$

Nonlinear Systems

- There exists a control policy which renders the dynamic ISS with respect to estimation error and system disturbances.

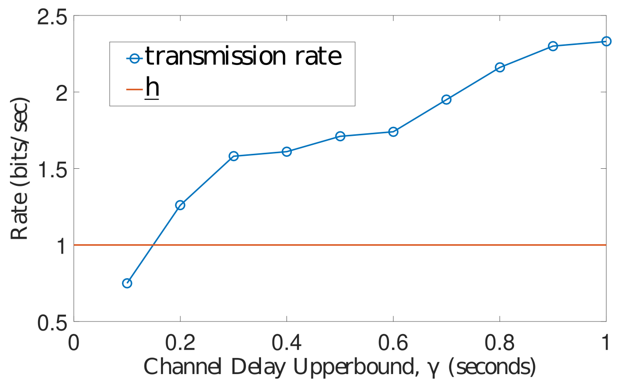
$$|X(t)| \le \beta'(|X(0)|, t) + \Pi'(|Z|_t) + \psi'(|W|_t)$$

$$\beta' \in \mathcal{KL} \qquad \Pi' \in \mathcal{K}_\infty(0) \qquad \psi' \in \mathcal{K}_\infty(0)$$

$$Z(t) = X(t) - \hat{X}(t)$$

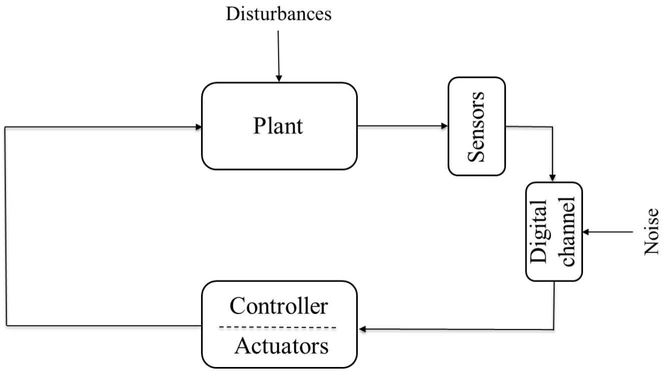
$$|W|_t = \sup_{s\in[0,t]} |W(s)|$$

  - Similar to linear plant
    - For small delay, we are below data-rate theorem
      - Kh, Hedayatpour, Franceschetti- 19
  - Extension to vector system
  - Relaxing the above assumption
    - Similar to Hespanha, Liberzon, Teel - 08 for periodic control



Uplink and Downlink channels

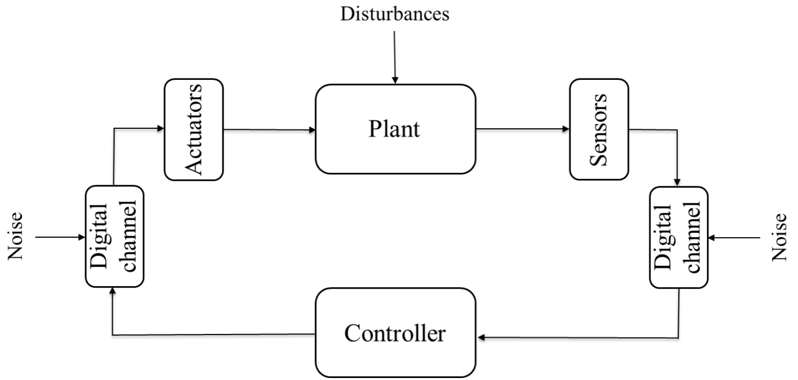
- Data-rate theorems focused on Uplink
  - Main bottleneck in mobile robots
    - Week on-board transmitter
  - Controller is co-located with the actuators
  - Serve as causal feedback
    - Acknowledge the received symbol to the sensor
    - Plant is the communication medium
      - Communication via control input



Uplink and Downlink channels

- A digital channel in the downlink, between the controller and the plant

  – Extension of theses data-rate results



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