SYNCHRONIZATION AND STATE ESTIMATION OF NONLINEAR SYSTEMS UNDER COMMUNICATION CONSTRAINTS

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1. INTRODUCTION

Communication network may cause a degradation of the control system performance through:

- **quantisation errors,**
- **transmission time delays,**
- **dropped measurements.**

Limitations of estimation and control under information constraints:

1. ***Wong, Brockett,* "Systems with finite communication bandwidth constraints," *IEEE TAC*, vol. 42, no. 9, 1997.**
2. ***Nair, Evans,* "Exponential stabilisability of finite-dimensional linear systems with limited data rates," *Automatica*, vol. 39, 2003.**
3. ***Tatikonda, Mitter*, "Control under Communication Constraints." *IEEE TAC,* vol. 49, no. 7, 2004.**
4. ***Nair, Fagnani, Zampieri, Evans,*"Feedback control under data rate constraints: an overview," *Proc. IEEE*, vol. 95, no. 1, 2007.**
5. ***Matveev, Savkin,* Estimation and Control over Communication Networks, Birkhauzer, Boston, 2009.**
6. ***Andrievsky, Matveev, Fradkov,* Control and estimation under information constraints: Toward a unified theory of control, computation and communications. Autom. Remote Control, vol. 71, no. 4, 2010.**

Data-Rate Theorem

[*G. Nair, R.J. Evans*, Automatica, 2003, 585-593.]

INFORMATION MUST BE TRANSPORTED AS FAST AS THE SYSTEM GENERATES IT, OR ELSE INSTABILITY OCCURS

$$x_{k+1} = Ax_k + Bu_k, \quad y_k = Cx_k,$$

$$Z = \{0, 1, \ldots, \mu - 1\}, \quad R = \log_2 \mu$$

$$s_k = \gamma_k(y_k, s_{k-1}) - \textbf{coder}$$

$$u_k = \delta_k(s_{k-1}) - \textbf{controller}$$

ρ - **exponential stabilizability of the system:**

$$\rho^{-kr} \|x_k\|^r \to 0 \text{ as } k \to \infty \Leftrightarrow$$

$$R > \sum_{|\eta_j| \geq \rho} \log_2 \left| \frac{\eta_j}{\rho} \right|$$

2. PASSIFICATION METHOD

2.1 Passification Problem

LTI single-input multiple-output system:

$$\dot{x} = Ax + Bu, \quad z = Cx, \qquad (1)$$

$x = x(t) \in \mathbb{R}^n$ — state vector, $u = u(t) \in \mathbb{R}^1$ — control, $z = z(t) \in \mathbb{R}^l$ — measured output, A, B, C — real matrices

Let G be $(1 \times l)$-matrix.

Passification problem:

find $(l \times 1)$-matrix K s.t. the closed loop system with $u = -K^{\mathrm{T}} z + v$ is strictly passive with respect to $\sigma = Gz$:

for some $\rho > 0$ and all $T > 0$

$$\int_0^T (\sigma v - \rho |x|^2)\, \mathrm{d}t \geq 0$$

holds for all trajectories of (1) starting from $x(0) = 0$.

2.2 Hyper Minimum Phase systems

Definition 1 *System (1) is called* minimum phase *with respect to the output* $\sigma = Gz$, *if the polynomial*

$$\varphi_0(s) = \det \begin{bmatrix} sI_n - A & -B \\ GC & 0 \end{bmatrix} \qquad (3)$$

is Hurwitz; hyper minimum phase *(HMP), if it is minimum phase and* $GCB > 0$.

2.3 Passification Theorem (Feedback KYP Lemma)

[*Fradkov*. Aut.Rem.Control (1974), Sib.Math.J. (1976), Europ.J.Control, 2003]

The following statements are equivalent:

(A1) There exist a positive definite $(n \times n)$-matrix H and $(l \times 1)$-matrix K s.t.

$$H(A+BK^{\mathrm{T}}C)+(A+BK^{\mathrm{T}}C)^{\mathrm{T}}H<0, \qquad (4)$$

$$HB=C^{\mathrm{T}}G^{\mathrm{T}}$$

(B1) System (1) is HMP w.r.t. $\sigma = Gz$.

(C1) There exists a feedback $u = K^{\mathrm{T}}z + v$ (5)

s.t. (1), (5) *is strictly passive w.r.t.* $\sigma = Gz$.

Rem. K in (4) can be found in the form $K = -\kappa G^{\mathrm{T}}$ where κ is a sufficiently large positive real number.

3. SYNCHRONIZATION AND STATE ESTIMATION OVER THE LIMITED-BAND COMMUNICATION CHANNEL

3.1 Coding Procedure

- **Static coder** $$q(y, M) = M \operatorname{sign}(y)$$
- **Coder range** $$M[k] = (M_0 - M_\infty)\rho^k + M_\infty$$
$k = 0, 1, 2, \dots$
- **Central number** $$c[k+1] = c[k] + \bar{\partial} y[k], \quad c[0] = 0$$
- **Deviation signal** $$\partial y[k] = y[k] - c[k]$$
- **Transmitted signal** $$\bar{\partial} y[k] = q(\partial y[k], M[k])$$

3.1 Controlled Synchronization of Passifiable Lur'e Systems

Master-slave synchronization of two identical nonlinear systems, modeled in the Lur'e form:

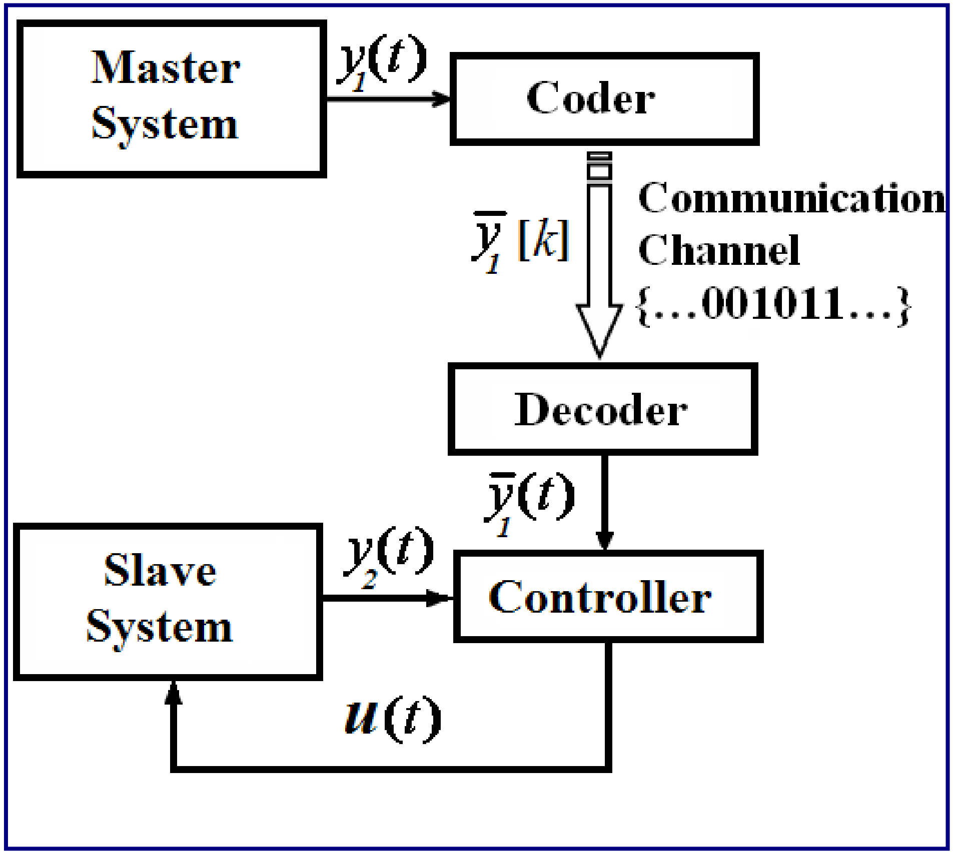
$$\dot{x}(t) = Ax(t) + B\psi(y_1), \;\; y_1(t) = Cx(t), \tag{9}$$

$$\dot{z}(t) = Az(t) + B\psi(y_2) + Bu, \;\; y_2(t) = Cz(t), \tag{10}$$

$x(t)$, $z(t)$ — n-dimensional vectors of state variables;
$y_1(t)$, $y_2(t)$ — scalar outputs; $u(t)$ — scalar control;
$\psi(y)$ — continuous nonlinearity, acting in span of control.

(9) – *master* (leader) system,
(10) – *slave* system (follower)

3.1.1 Transmission of the output signal of the master system



Controller – the static output feedback:

$$u(t) = -K\bar{\varepsilon}(t), \qquad (12)$$

$\bar{\varepsilon}(t) = y_2(t) - \bar{y}_1(t)$, K is a scalar controller gain.

Transmission of the output signal (cont.)

The Lyapunov function $V(e) = e^{\mathrm{T}}Pe$ and the gain K s.t.

$$\dot{V}(e) \le -\mu V(e)$$

for some $\mu > 0$ and $\delta_y(t) = 0$ exist if and only if

$$W(\lambda) = C(\lambda I - A)^{-1}B \text{ is HMP,}$$

where $e(t) = x(t) - z(t)$, $\delta_y(t) = y_1(t) - \bar{y}_1(t)$.

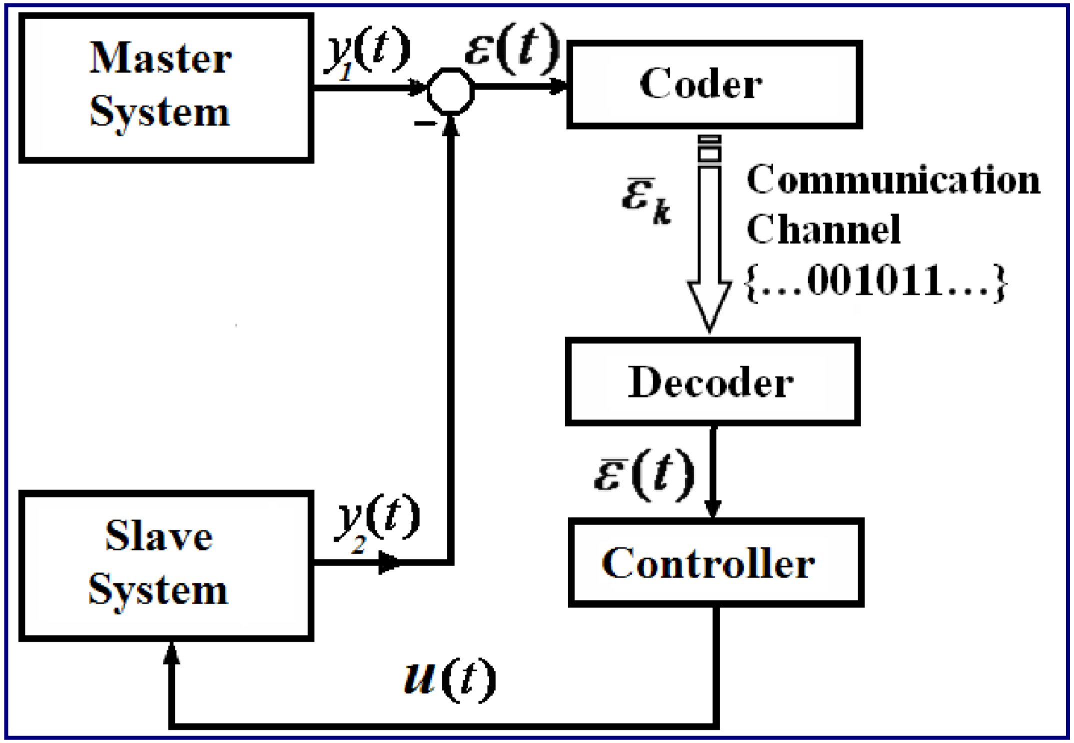
Limit synchronization error $\lim_{t\to\infty} \|e(t)\| \le C_e \Delta$,

where $C_e = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\frac{L_\psi + |K|}{\mu}$, Δ – upper bound on the transmission error, $L_\psi > 0:\ |\psi(y) - \psi(y+\delta)| \le L_\psi |\delta|$.

$\Delta = \beta L_y / R$, where $\beta \approx 1.688$, $L_y = \sup_{x\in\Omega} |C\dot{x}|$.

➔ Limit synchronization error is inversely proportional to the channel capacity ***R*** (FAE, Phys.Rev.E, 2006)

3.1.2 Transmission of the error between systems' outputs



$$u(t) = -K\bar{\varepsilon}(t),$$

$$\text{where } \bar{\varepsilon}(t) = \bar{\varepsilon}[k] \text{ as } t_k < t < t_{k+1},$$

$$\bar{\varepsilon}[k] = M[k]\,\mathrm{sign}(\varepsilon(t_k))$$

Transmission of the error (cont.)

A1. Nonlinearity $\psi(y)$ is Lipschitz continuous:

$$|\psi(y_1) - \psi(y_2)| \le L_\psi |y_1 - y_2|$$

for all y_1, y_2 and some $L_\psi > 0$.

A2. The linear part of (9) is strictly passifiable.

Then, as follows from the Passification Theorem,

$\exists\ P = P^{\mathrm{T}} > 0$ and K s.t. for any η: $0 < \eta < \eta_0$:

$$PA_K + A_K^{\mathrm{T}} P \le -2\eta P,\ \ PB = C^{\mathrm{T}}, \qquad (17)$$

where $A_K = A - BKC$, $\eta_0 -$ stability degree of $\beta(\lambda)$

Any sufficiently large K can be chosen.

Transmission of the error (cont.)

Theorem (FAE, TCAS-2009)

- A1, A2 hold;
- K satisfies passivity relations (17);
- the coder parameters ρ, T:

$$e^{\eta T}(e^{L_F T}-1) \le \frac{L_F}{\|C\|(K\|B\|+L_F)}, \; e^{-\eta T} < \rho < 1,$$

where $L_F = \|A\| + L_\psi \|B\| \cdot \|C\|$, η is from (17).

- The coder range $M[k] = M_0 \rho^k$.

<u>Then</u> $\forall e(0)$ s.t. $e(0)^{\mathrm{T}} P e(0) \le M_0^2$:
$e[k]$ decays exponentially: $|\varepsilon[k]| \le \|e[k]\| \le M_0 \rho^k$
and $|\varepsilon(t)| \le |\varepsilon[k]|$ for $t_k \le t < t_{k+1}$.

3.1 State Estimation of Passifiable Lur'e Systems

Nonlinear observer, embedded to the coder:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\psi(\hat{y}) + L\bar{\varepsilon}(t), \;\; \hat{y}_1(t) = C\hat{x}(t), \tag{20}$$

where $\hat{x}(t) \in \mathbb{R}^n$; $\hat{y}_1(t) \in \mathbb{R}^1$; error $\varepsilon(t) = y_1(t) - \hat{y}_1(t)$;

$\bar{\varepsilon}[k] = M[k]\,\mathrm{sign}(\varepsilon(t_k)),$ $\bar{\varepsilon}(t) = \bar{\varepsilon}[k]$ as $t \in [t_k, t_{k+1}]$

$L -$ observer gain (design parameter).

$\bar{\varepsilon}[k]$ is transmitted over the channel.

Observer at the receiver side:

$$\dot{\hat{x}}_d(t) = A\hat{x}_d(t) + B\psi(\hat{y}_d) + L\bar{\varepsilon}(t), \;\; \hat{y}_d(t) = C\hat{x}_d(t),$$
$$\hat{x}_d(0) = \hat{x}(0).$$

Convergence conditions follow from (FAE, TCAS-2009)

3.2 Adaptive Synchronization of Passifiable Lur'e Systems

3.2.1 Problem Statement and Synchronization Scheme

Nonlinear uncertain system ("transmitter", "master system"):

$$\dot{x} = Ax + \psi_0(y) + B\sum_{i=1}^{m}\theta_i\psi_i(y), \quad y = Cx, \qquad (22)$$

x - transmitter state *n*-vector; *y* – *l*-vector of outputs (to be transmitted over the communication channel);

$\theta = [\theta_1, \ldots, \theta_m]^{\mathrm{T}}$ - parameters.

Assumption: $\psi_i(\cdot)$, *A*,*C*, *B* are known; only *y*(*t*) can be measured.

To achieve synchronization between two chaotic systems: *adaptive observer* [*Fradkov, Nijmeijer, Markov, Int. J. Bifurc. Chaos*, 10 (12), pp. 2807, 2000].

3.2.1 Adaptive observer

- *Tunable observer*:

$$\dot{\hat{x}} = A\hat{x} + \psi_0(\bar{y}) + B\sum_{i=1}^{m}\hat{\theta}_i\psi_i(\bar{y}) + L(\bar{y}-\hat{y}), \quad \hat{y} = C\hat{x}, \quad (24)$$

x –observer state *n-dim.* vector, *y* – observer output *l-dim.*vector,
$\hat{\theta}_i$ – tunable parameters (*i* = 1,2,...,*m*).

- *Adaptiation algorithm*:

$$\dot{\hat{\theta}}_i = -\gamma(\bar{y}-\hat{y})\psi_i(\bar{y}) - \alpha\hat{\theta}_i, \quad i = 1, 2, \ldots, m, \quad (25)$$

$\gamma > 0 - adaptation\ gain\,; \ \alpha \geqslant 0 - regularization\ gain.$

3.2.2 Performance evaluation

Theorem. Let the following assumptions hold:

A1. The observer gain matrix L is such that the transfer function $W_L(\lambda) = C(\lambda\mathbf{I} - A + LC)^{-1}B$ is strictly passive.

A2. The system (22) possesses a bounded invariant set $\Omega_\theta \subset \mathbb{R}^n$ for any $\theta \in \Theta \subset \mathbb{R}^m$, where Θ is the set of possible values of uncertain parameters and $x(0) \in \Omega$.

A3. Functions $\psi_i(y)$, $i = 0, 1, \dots, m$ are bounded and Lipschitz continuous in the closed Δ-vicinity of Ω_θ, i.e. $|\psi_i(y)| \le L_\psi$, $|\psi_i(y') - \psi_i(y)| \le L'_\psi$ for some L_ψ, L'_ψ and for all $y = Cx$, $x \in S_\Delta(\Omega_\theta)$, where $S_\Delta(\Omega_\theta) = \{x : \exists z \in \Omega_\theta : \|x - z\| \le \Delta\}$.

Then there exist constants $C_1 > 0$, $C_2 > 0$ such that for any $\Delta > 0$ the choice of design parameters $\alpha = \Delta^2$, $\gamma = C_2/\Delta^2$ guarantees that the synchronization goal $Q \le \Delta_x$ is achieved for $\Delta_x = C_1\Delta$, i.e. the limit synchronization error Δ_x is proportional to the transmission error Δ.

3.2.3 HMP condition in observer design problem

According to the observer version of the passification theorem by Efimov and Fradkov (2006), the vector L satisfying assumption *A1* exists if and only if the transfer function $W(\lambda) = C(\lambda\mathbf{I} - A)^{-1}B$ is HMP. To find vector L satisfying *A1* under the HMP condition it is sufficient to choose L in the form $L = -\kappa C$, where $\kappa > 0$ is large enough.

Results are extended to the case of bounded disturbances (Fradkov A.L., Andrievsky B., Ananyevskiy M.S. Passification based synchronization of nonlinear systems under communication constraints and bounded disturbances. Automatica, V. 55 (5), 2015, pp. 287–293).

CONCLUSIONS

- A unified exposition of passification-based approach for synchronization and state estimation of nonlinear systems over the limited-band communication channel is given.

- Relevance of passifiability condition for the posed problems is demonstrated.

- Future research is aimed at taking into account signal drops and delays in the communication channel.

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