OUTLINE

Preliminary. Adaptive Coding Procedure

At $k = 0, 1, \ldots$ coder forms *deviation*: $\partial y_k = y_k - c_k$, discretized with a given $M = M_k$: $\bar{\partial} y_k = q(\partial y_k, M_k)$, transmitted over the channel to the decoder, where *centroid* $c_k = c_{k-1} + \bar{\partial} y_k$, $c_0 = 0$.
Quantizer range M_k is chanded:

$$\begin{aligned} &\sigma_k = \operatorname{sign}(\bar{\partial} y_k), \\ &\lambda_k = (\sigma_k + \sigma_{k-1} + \sigma_{k-2})/3, \quad \lambda_0 = 0, \\ &M_{k+1} = m + \begin{cases} \rho M_k, & \text{as } |\lambda_k| \leqslant 0.5, \\ M[k]/\rho, & \text{otherwise}, \end{cases} \end{aligned} \tag{1}$$

$0 < \rho \leqslant 1$ – decay parameter; $M[0] = M_0$, m – lower bound.
The decoder calculates the variables $\tilde{c}_k$, $\tilde{M}_k$ based on received codeword flow similarly to c_k, M_k.

Physical examples

Remote state estimation scheme was studied using the experimental data for:

- *Multipendulum Mechatronic Set-up* (MMS);
- Quanser–LAAS Helicopter benchmark;
- Adaptive Coding Procedure Examination Based on the In-flight Data From Quadrotors.
- State Estimation of Spatially Distributed Systems with Quantization of the Transmitted Data
- Experiments on controlled synchronization with three-computer setup [*Fradkov et. al.*, Automatica, 2015].

State estimation of the Multipendulum Mechatronic Set-up

Photo of the chain of twelve pendulum sections and the motor

Fradkov et. al. Mechatronics, 2012; — Eur. Phys. J., 2014.

State estimation of the Multipendulum Mechatronic Set-up: chain of 4 pendulums and the motor, attached via the spring to pendulum #1.

Modified state estimation scheme: exogenous plant model is taken instead of autonomous one:

$$\dot{x}(t) = Ax(t) + B\psi(y) + Du(t), \ y(t) = Cx(t), \qquad (2)$$

$u(t) \in \mathbb{R}^m$ – external input, D – $(n \times m)$-matrix.

Embedded observer:

$$\begin{aligned} &\dot{\hat{x}}(t) = A\hat{x}(t) + B\psi(\hat{y}) + D\tilde{u}(t) + L\bar{\varepsilon}(t), \ \hat{y}(t) = C\hat{x}(t), \\ &\bar{\varepsilon}(t) = \bar{\varepsilon}[k] \ \text{ as } \ t \in [t_k, t_{k+1}), \quad t_k = kT_s, \end{aligned} \qquad (3)$$

$\tilde{u}(t)$ – measured exogenous input $u(t)$.

State estimation of the Multipendulum Mechatronic Set-up (cont.)

Chain of 4 pendulums:

$$\begin{cases} \ddot{\varphi}_1 + \rho\dot{\varphi}_1 + \omega_0^2 \sin\varphi_1 - k(\varphi_2 - 2\varphi_1) = ku(t), \\ \ddot{\varphi}_2 + \rho\dot{\varphi}_2 + \omega_0^2 \sin\varphi_2 - k(\varphi_{i+1} - 2\varphi_2 + \varphi_1) = 0, \\ \ddot{\varphi}_3 + \rho\dot{\varphi}_3 + \omega_0^2 \sin\varphi_i - k(\varphi_4 - 2\varphi_3 + \varphi_2) = 0, \\ \ddot{\varphi}_4 + \rho\dot{\varphi}_4 + \omega_0^2 \sin\varphi_4 - k(\varphi_4 - \varphi_3) = 0, \end{cases} \tag{4}$$

φ_i – pendulum angles; u – motor rotation angle.
$\mu = 0.95\ \mathrm{s}^{-1}$, $\omega_0 = 5.5\mathrm{s}^{-1}$, $k = 5.8\ \mathrm{s}^{-2}$;
Sampling time $T_s \in [10, 100]$ ms.

$$q_\nu(y, M) = \begin{cases} \delta \cdot \langle \delta^{-1} y \rangle, & \text{if } |y| \leqslant M, \\ M \operatorname{sign}(y), & \text{otherwise}, \end{cases}$$

where $\delta = 2^{1-\nu} M$ — from $\nu = 0$ (binary coder) to $\nu = 8$.

State estimation of the Multipendulum Mechatronic Set-up (cont.)

Two coding schemes have been implemented:

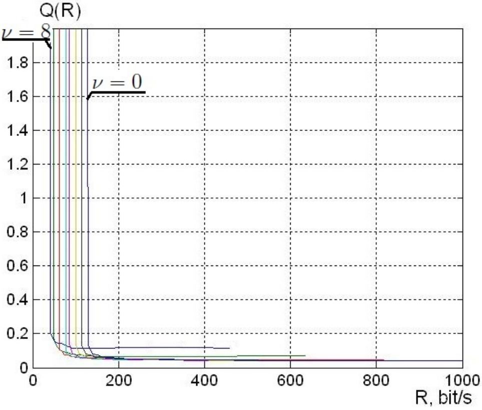
1. the first-order coder – for transferring the motor rotary angle,
2. the full-order coder with embedded observer – for transferring pendulum rotary angle.

Measured data have been processed by means of the data transmission procedures. The estimation errors have been calculated.

Bit-per-second rate for each channel: $R = \nu/T_s$.

State estimation of the Multipendulum Mechatronic Set-up (cont.)

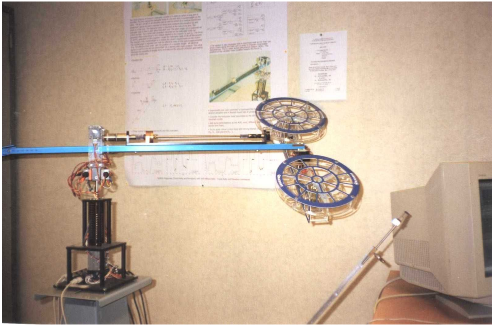
Mean-square relative transmission error Q v.s. transmission bitrate R (bit/s) for different ν.



– the binary coder is optimal in the sense of R;
– for full order coder there exists a threshold which limits the admissible bit-rate for data transmission.

State Estimation for Quanser–LAAS Helicopter Benchmark

The "Helicopter": a base with a long arm, carriying the "body". The arm can tilt on an elevation axis and swivel on a vertical (travel) axis. Two motors with propellers are mounted on the helicopter body.



Andrievsky, Fradkov, Peaucelle, IEEE Trans. CST, 2010.

State Estimation for Quanser–LAAS Helicopter (cont.)

Adaptive coding procedure is employed.
At $t_k = kT_s$ $(k = 0, 1, \ldots)$ at the sensor node the *state estimation error* $e_k = \theta(t_k) - \hat{\theta}(t_k)$ is calculated and coded as $\sigma_k = \operatorname{sign} e_k$; σ_k is transmitted over the channel.
Quantizer range M_k:

$$\lambda_k = (\sigma_k + \sigma_{k-1} + \sigma_{k-2})/3,$$

$$M_{k+1} = m + \begin{cases} \rho M_k, & \text{as } \ |\lambda_k| \leqslant 0.5, \\ M[k]/\rho, & \text{otherwise.} \end{cases}$$

Decoder output $\bar{e}_k = M_k \sigma_k$ is expended to $[t_k, t_{k+1})$:

$$\bar{e}(t) = \bar{e}_k \quad \text{as} \ \ t \in [t_k, t_{k+1}), \tag{5}$$

where $t_k = kT_s$, $k = 0, 1, \ldots$.

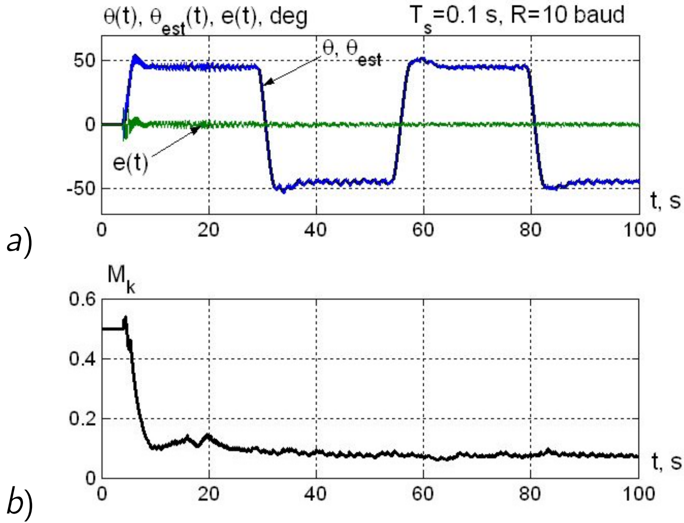
State Estimation for Quanser–LAAS Helicopter (cont.)

Embedded State Observer:

$$\begin{cases} \dot{\hat{\theta}}(t) = \hat{\omega}(t) + l_1 \bar{e}(t), \\ \dot{\hat{\omega}}_x(t) = -a_m^{\omega_x} \hat{\omega}_x(t) - a_m^{\theta} \sin\left(\hat{\theta}(t) - \theta_0\right) \\ \qquad\qquad + k_m^{v} \mu(t) + l_2 \bar{e}(t), \end{cases} \tag{6}$$

$\hat{\theta}(t)$, $\hat{\omega}_x(t)$ – estimates of the pitch $\theta(t)$ and pitch angular rate $\omega_x(t)$; l_1, l_2 – observer gains.
The data coding/decoding and state estimation procedures for the “Helicopter” pitch motion should be realized both at the transmitter (sensor) and at the receiver (controller) nodes of the system.

State Estimation for Quanser–LAAS Helicopter (cont.)



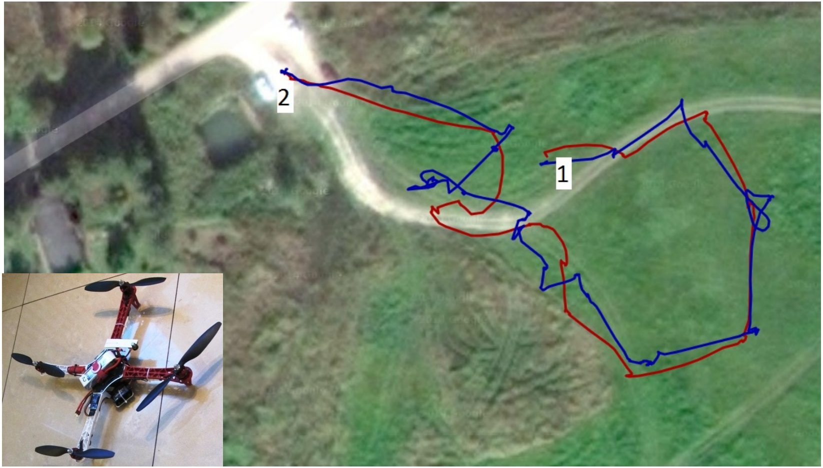
Adaptive coding procedure examination based on the in-flight data

During the experiment, two quadrotors have been employed, making up the leader-slave formation. The leading quadrotor has been governed by the commands from the remote operator's computer, the slave quadrotor has constantly followed the leader based on the leader's position information, obtained through a telemetry channel with a small data rate. The *GPS* receivers were used to obtain current quadrotors position. *Xbee-Pro 2.4GHz* modules have been used for data exchange between the quadrotors.

Fradkov et. al., IFAC-PapersOnLine 49-13 (2016) 275–280; 12th IFAC Workshop ALCOSP 2016.

Adaptive coding procedure examination (cont.)

Flight Test Area



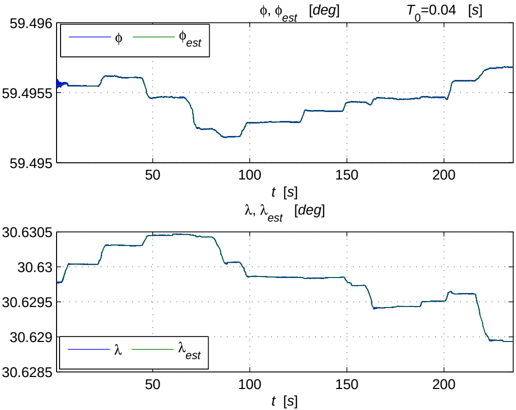
Adaptive coding procedure examination (cont.)

Adaptive coding procedure (1), with the binary coder $q(\sigma, M) = M\,\mathrm{sign}(\sigma)$, ($\sigma$ stands for the quadrotor latitude/longitude), and the embedded observer

$$\begin{cases} \bar{\sigma}_k = q(\sigma_k, M_k), \\ \hat{x}_{k+1} = \hat{x}_k + T_s\hat{V}_k + l_1\bar{\sigma}_k, \\ \hat{V}_{k+1} = \hat{V}_k + l_2\bar{\sigma}_k, \end{cases} \tag{7}$$

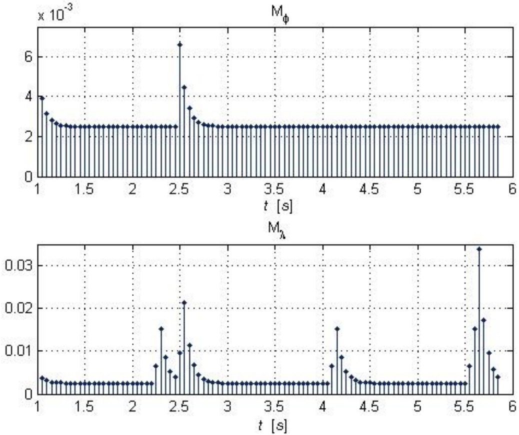
was examined based on the flight data. Parameters:
$M_0 = 5 \cdot 10^{-3}$, $m = 10^{-4}$.
$T_s \in \{0.01, 0.025, 0.04\}$ [s] for different simulation runs (data bitrate $R \in \{100, 40, 25\}$ [bit/s]); decay parameter $\rho = \exp(-15T_s)$; observer matrix L: ensuring eigenvalues $z_{1,2} = \exp(\lambda_{1,2}T_s)$, for $\lambda_{1,2} = -0.35 \pm 0.36i$.

Time histories of measured and estimated leader coordinates φ, λ; $T_s = 0.04$ s ($R = 25$ bit/s).



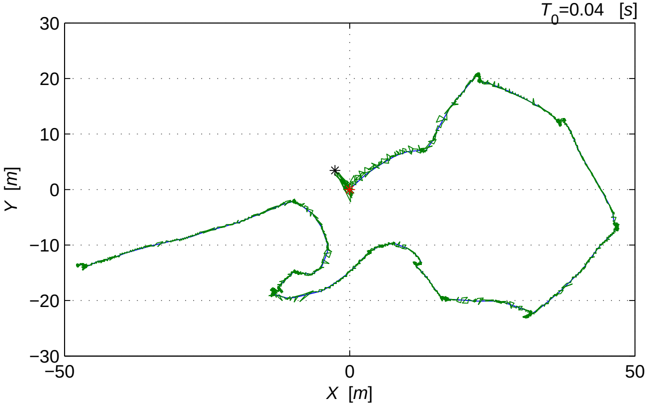
Numerical study of adaptive coding based on flight test

Adaptive adjustment of φ and λ coder ranges. $T = 0.04$ s ($R = 25$ bit/s).



Numerical study of adaptive coding based on flight test

Actual and estimated leader trajectories on the horizon plane. $T = 0.04$ s ($R = 25$ bit/s).



State estimation of spatially distributed systems with quantization

One-dimensional spatially distributed system:

$$z_{tt}(t,x) - z_{xx}(t,x) + z(t,x) = 0, \quad t \geqslant 0, \quad x \in [0,1] \quad (8)$$

initial conditions: $z(0,x) = z^0(x)$, $z_t(0,x) = z^1(x)$,
boundary conditions: $z(t,0) = z_x(t,1) = 0$.
measured output $y(t) = z(t,l)$ at some inner point $l \in (0,1)$.

Problem: estimation of $z(t,x)$ based on sampled and quantized measurements of $y(t)$.

Andrievsky B. Proc. 2016 IEEE Conference on Norbert Wiener in the 21st Century, 21CW 2016.

State estimation of spatially distributed systems (cont.)

Approximation of PDEs as a linear finite-dimentional system by uniform splitting $[0,1]$ into K sub-intervals:

$$\dot{\zeta}(t) = A\zeta(t), \quad y(t) = C\zeta(t), \quad \zeta(0) = \zeta^0, \quad t \geqslant 0, \qquad (9)$$

$\zeta \in \mathbb{R}^{2K}$ – state-space vector, K – number of points inside interval $x \in [0,1]$; $A \in \mathbb{R}^{2K\times 2K}$,
$C = [0,\ldots,1,\ldots,0] \in \mathbb{R}^{1\times 2K}$.
Denote $h_K = K^{-1}$ – *discretization step*, $i = 1,\ldots,K-1$.

$$z_x(t,x_i) \approx \big(z(t,x_{i+1}) - z(t,x_{i-1})\big)/(2h),$$
$$z_{xx}(t,x_i) \approx \big(z(t,x_{i+1}) - 2z(t,x_i) + z(t,x_{i-1})\big)/h^2,$$
$$z(t,x_0) = z(t,x_1), \quad z(t,x_K) = z(t,x_{K-1}).$$

State estimation of spatially distributed systems (cont.)

Denote time derivatives $z_t \in \mathbb{R}^k$ as $v = z_t$; $z_i(t) \equiv z(t, x_i)$, $v_i(t) \equiv z_t(t, x_i)$, $i = 1, \ldots, k$ for the corresponding state variables of the discretized system $\Longrightarrow$ linear model $\ddot{z}(t) = A_z z(t)$ with

$$A_z = \frac{1}{h^2} \begin{bmatrix} -1 & 1 & 0 & \ldots & 0 \\ 1 & -2 & 1 & \ldots & 0 \\ 0 & 1 & \ddots & \ldots & \vdots \\ 0 & \ldots & \ldots & -2 & 1 \\ 0 & \ldots & 0 & 1 & -1 \end{bmatrix}. \qquad (10)$$

State estimation of spatially distributed systems (cont.)

In terms of the state vector $\zeta = \mathrm{col}\{x, v\} \in \mathbb{R}^{2k}$, state-space representation (9) has the following block matrix

$$A = \begin{bmatrix} \mathbf{0}_{K\times K} & \mathbf{I}_K \\ A_z & \mathbf{0}_{K\times K} \end{bmatrix}, \qquad (11)$$

$\mathbf{0}_{K\times K}$ – $(K \times K)$-zero matrix, matrix A_z is inherited from (10), $\mathbf{I}_K$ – identity matrix of order K.

State estimation of spatially distributed systems (cont.)

For data transmission, the full-order coding-decoding procedure with embedded observer was used

$$\dot{\hat{\zeta}}(t) = A\hat{\zeta}(t) + L\bar{\varepsilon}(t),\ \hat{y}(t) = C\hat{\zeta}(t) \qquad (12)$$

on the coder side, where $\bar{\varepsilon}[k] = M[k]\,\mathrm{sign}(\varepsilon(t_k))$, and follower

$$\dot{\hat{x}}(t) = A\hat{x}(t) + L\bar{\varepsilon}(t),\ \hat{y}_1(t) = C\hat{x}(t),$$

Adjustment of coder range $M[k]$ is governed by adaptive zooming procedure (1). Vector $\hat{\zeta}(t)$ serves as the estimate of plant (8) state variables $z(t, x_i)$, $z_t(t, x_i)$ in specified nodes $x_i = i \cdot h_K$, $i = 1, \ldots, K-1$.

State estimation of spatially distributed systems (cont.)

Simulation results

For simulations, PDEs are discretized in $x \in [0,1]$ by splitting $[0,1]$ into $N \gg K$.

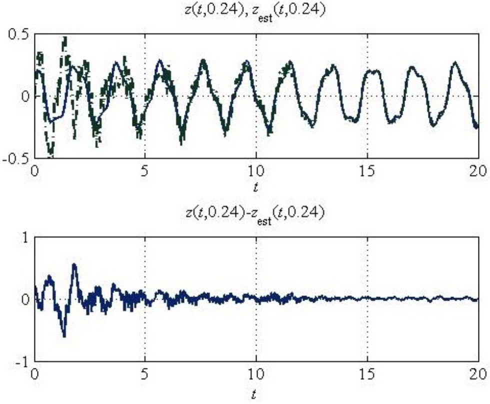
Resulting system of $N-1$ ODEs of second order numerically solved over a time interval $[0,T]$ by standard MATLAB routine *ode45*.

Parameters: $K = 21$, $N = 2501$;
L – by pole placement, s.t. $\lambda_L = \lambda_A - 0.25$ (λ_L and λ_A – spectra of matrices $A - LC$ and A).
Adaptive zooming (1): $M[0] = 0.5$, $\rho = \exp(-10T_s)$, $m = 0$.
Sampling period $T_s \in [0.005, 0.05]$ s for different runs.

State estimation of spatially distributed systems (cont.)

$l = 0.24$, $R = 50$ bit/s.

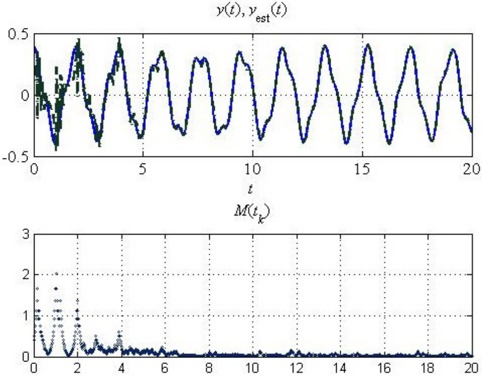
- $z(t, l)$ – solid line, estimate $\hat{z}(t, l)$ – dash-dot line;
- estimation error (lower plot)



State estimation of spatially distributed systems (cont.)

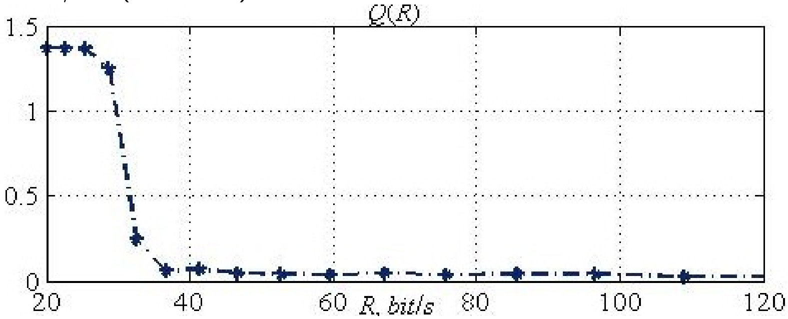
$R = 50$ bit/s.

- measured output $y(t)$ – solid line, estimate $\hat{y}(t)$ – dash-dot line;
- range $M(t_k)$ of adaptive coder (1) (lower plot).



State estimation of spatially distributed systems (cont.)

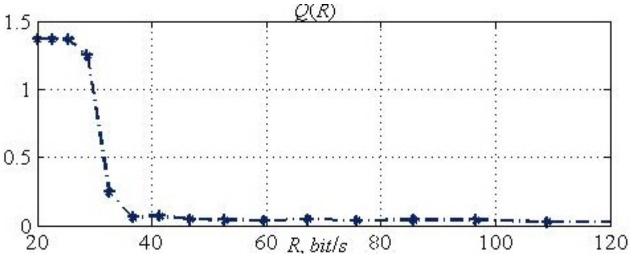
Generalized accuracy index $Q = Q(R)$ – maximum of the estimation error magnitude at the interval $t \in [20, 25]$ vs $R = 1/T_s$ ($l = 0.24$).



State estimation of spatially distributed systems (cont.)

Corollary

- After some transient time (≈ 15 s) the estimation error falls to the 10% range in magnitude of recovered process $z(t, l)$.
- $M[k]$ automatically increases during the transients (the "*zoom-out*" stage) and decreases in the steady-state mode (the "*zoom-in*" stage).
- For this example the limited data rate, when state estimation is possible, is about 35 bit/s.
- Accuracy does not increase on R if $R > 80$ bit/s. It is explained by discretization of (8) on spatial variable x.


Q(R)
1.5
1
0.5
0
20
40
60
R, bit/s
80
100
120

Conclusions

- A unified exposition of passification-based approach for synchronization and state estimation of nonlinear systems over the limited-band communication channel is given.
- Relevance of passifiability condition for the posed problems is demonstrated.
- Experimental results for various physical systems are provided, demonstrating the practical applicability of the theoretical statements.
- Future work: investigations on state estimation and synchronization of nonlinear physical systems over the limited bandwidth communication channel with time delays, data corruption and losses.

SYNCHRONIZATION AND STATE ESTIMATION OF NONLINEAR PHYSICAL SYSTEMS UNDER COMMUNICATION CONSTRAINTS

Alexander Fradkov, Boris Andrievsky
IPME RAS & St. Petersburg University
Saint Petersburg, Russia
fradkov@mail.ru; boris.andrievsky@gmail.com

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